ADDIS ABABA SCIENCE AND TECHNOLOGY UNIVERSTIY

COLLEGE OF ARCHITECTURE AND CIVIL ENGINEERING

DEPARTMENT OF CIVIL ENGINERING

(GEOTECHNICAL ENGINEERING)

# COMPARISION OF RELIABILITY BASED DESIGN WITH DETERMINISTIC APPROACH IN GEOTECHNICAL ENGINEERING PROBLEMS

A Thesis Submitted to the Department of Civil Engineering, College of Architecture and Civil Engineering in Partial Fulfillment of the Requirement for a Master of Science Degree in Geotechnical Engineering

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# ADDIS ABABA SCIENCE AND TECHNOLOGY UNIVERSTIY

## APPROVED BY BOAED OF EXAMINERS

## APPROVAL PAGE

This MSc thesis Proposal entitled with **"COMPARISION OF RELIABILITY BASED DESIGN WITH DETERMNISTIC APPROACH IN GEOTECHNICAL ENGINEERING"** has been approved by the following examiners after the Thesis presentation for the Master of Science in Geotechnical Engineering.

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## Declaration

Hereby declare that the work which is being presented in this thesis entitles "**Comparison of Reliability Based Design with Deterministic Approach in Geotechnical Engineering Problems**" is original work of my own, has not been presented for a degree in any university and that all sources of materials used for this thesis duly acknowledge.

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Symbols Definition

$Qu$ = Ultimate bearing capacity

$Qall$ = Allowable bearing capacity

$c$ = Soil cohesion

$\gamma$ = Soil unit weight

$B$ = Footing breadth

$q$ = Overburden pressure

$Df$ = Depth of footing

$Nc$ = Cohesion factor

$Nq$ = Surcharge factor

$N\gamma$ = Unit weight factor

$\gamma L$ = Load factor for live load

$\gamma D$ = Load factor for dead load

$\beta$ = Reliability index

$p_f$ = Probability of failure

$p_s$ = Probability of survival

$\varphi$ = Statistics resistance factor

$\phi$ = Friction angle

$Qh$ = Horizontal component of inclined load

$Qv$ = Vertical component of inclined load

$\mu qu$ = Mean bearing capacity

$\sigma$ qu = Standard deviation of bearing capacity

$\nu$qu = Skewness

$\rho c'\tan\phi'$ = Correlation coefficient between cohesion & angle of friction

$\xi$ = Standard unit

R = Resistance capacity

Qi = Load effects

M = Moment

Af = Effective contact area of footing

Ca = Unit adhesion on base of footing

Si = Shape factor

ii = Inclination factor

di = Depth factor

bi = depth factor for Euro code

$\rho$ = Density

fcd =Compressive design strength of concrete

fctd = Tensile design strength of concrete

fyd = Yield strength of steel

$\omega$ = Moisture content

$\nu$ = Poissons ratio

$R_m$ = Measured nominal resistance

$R_n$ = Predicted nominal resistance

$\lambda$i = Bias factor

$\eta_i$ = Load modifier

D = Pile diameter

E = Young's modulus

e = Eccentricity

eB, eL = Eccentricity of load resultant with respect to centroid of footing

$A_p$ = Single pile bearing tip area

$A_s$ = Single pile shaft area

$A_{pg}$ = Gross bearing tip area of piles

$A_{sg}$ = Gross shaft area of piles

$q_p$ = Pile bearing tip capacity

$q_s$ = Pile shaft capacity

$Q_p$ = Pile bearing tip capacity

$Q_s$ = Pile shaft capacity

$Q_{pg}$ = Gross piles bearing tip capacity

$Q_{sg}$ = Gross piles shaft capacity

k1 = Factor for shape of foundation

k2 = Factor for shape of foundation

# Abstract

Geotechnical engineers face challenges to be certain during foundation design due to different sources of uncertainty of soil properties. These uncertainties include inherent uncertainty, measurement uncertainty and transformation uncertainty. Classical determistic methods such as by Terzaghi, Meyerhof, Hansen, Vesic, etc. equations are commonly used for calculation of bearing capacities. These methods use single factor of safety to overcome exististing uncertainities. This factor of safety includes uncertainties appearing on properties of soils without considering uncertainties due to loads. To overcome these problems reliabilty based design methods are analyzed and compared with deterministic methods. Reliability based design methods used here are First Order Second Moment(FOSM) method, Second Order Second Moment (SOSM) method and Point Estimation Method(PEM). FOSM and SOSM methods did not allow the evaluation of the skewness coefficient, thus being substantially less accurate than the Point Estimate Method. It is also observed from numerical expressions that using correlated random variables give more reliable values than uncorrelated ones. During design of isolated spread footing it is obtained that deterministic approaches give 60.6cm effective depth of footing and 28 ϕ14 bars that weigh 243.78kg which will cost 10,967 birr while reliability methods give a depth of 65.6cm which requires 30 ϕ14 bars (262.44kg) with 11,808birr cost. According to this reliability method gives higher depth and requires higher cost i.e footing depth variation shows 8.25% and cost of reinforcement shows 7.67% difference between the two methods. In the other case, during pile foundation design using reliability methods give 12 piles while deterministic approaches give 16 piles . This deals reliability methods give less number of piles than deterministic approaches. It is observed from carried out results that using RBD approaches increase confidences and also decreases un necessary additional construction costs. Generally, reliability based design methods play their great role on safe and economical design of geotechnical structures and it is better to use reliability methods as complementary ways to deterministic approaches and in turn use deterministic approaches as bench marks to reliability based design methods.

# CHAPTER ONE

# 1 INTRODUCTION

## 1.1.General background

There are different problems in geotechnical engineering. Among several problem some of them appear due defects and challenges appear during design of foundation, retaining walls, bridges, slope stability. This paper mainly focuses on foundations. Foundations are designed to transmit the load of a structure to the soil on which it is resting without overstressing it. Overstressing the soil can result in either excessive settlement or shear failure of the soil, both of which could damage the structure. Thus the bearing capacity of soils must be evaluated to avoid these problems. According to this, foundations are designed to bear loads from upper structures, including vertical and horizontal loads, which are combinations of dead and live loads. Live loads vary much more during the life of a foundation comparing to dead load. Live load can be vertically, such as machinery load, human weight and earthquake load, and horizontally, such as wind and earthquake load. Dead load can also be inclined such as load on foundations of bridge abutments. The combination of live and dead loads, or horizontal and vertical loads, result in the variation of magnitude and direction of loading imposed on a foundation. In geotechnical engineering, allowable bearing capacity is obtained by dividing ultimate bearing capacity with factor of safety. This factor of safety considers uncertainties appear in soil properties. To account for the variation of loading, a reliability analysis based on probabilistic theory is required.

### 1.1.1 Deterministic methods

In deterministic methods all data is known beforehand. Deterministic geotechnical analyses use a single factor of safety, which implicitly includes all sources of variability and uncertainty inherent in geotechnical design. This type of calculation of allowable bearing capcity is also known allowable stress design method. One of the most commonly used deterministic equations for bearing capacity analysis is the Terzaghi bearing capacity equation. Terzaghi's ultimate bearing capacity equation is then divided by a factor of safety to give allowable loading levels for design. Terzaghi's equation has a limitation of not accounting for inclined loads. Due to this Meyerhof, Hansen and Vesic developed bearing capacity equation by considering the effect of inclined loads.

### 1.1.2 Load and resistance factor design (LRFD) method

Load and resistance factor design introduce reliability-based design benefits into geotechnical designs, e.g. increased construction economies for low failure consequence (low risk) problems, increased investigation effort, etc. This method replaces single factor-of-safety with a set of partial safety factors (load and resistance factors) acting on individual components of resistance and load. It also accounts for load and resistance uncertainties.

Load and resistance factors are derived to account for:- variability in load and material properties, variability in construction and model error failure consequences. Two common resistance factors are implemented; Total resistance factor which is a single resistance factor applied to the final computed soil resistance and partial resistance factors which are multiple resistance factors applied to components of soil strength separately.

### 1.1.3 The factor of safety

The factor of safety is commonly defined as the ratio of the resistance of the structure (Rn) to the load effects (Q) acting on the structure.Traditional deterministic approach is based on the concept of the safety factor (FS), which is defined as the ratio between values of available strength or, more generally, the resistance R to failure and the load Q soliciting the failure of an engineering system. Typical values of the safety factor commonly adopted in the geotechnical field are, for example, FS=2.5-3.0 for the bearing capacity problem or FS=1.5 for the slope stability design of new earth dams.

Unfortunately this conventional analysis leads to conservative designs because uncertainties in analysis parameters are not taken into account during the calculation of the safety factor. In this sense the factor of safety is not a sufficient indicator of safety because the uncertainties in material and load properties can significantly influence the probability of failure.

### 1.1.4 Reliability based design methods

Engineering community, building users and owner of construction project always expects structure and its foundation to be designed with a reasonably safety margin. In practices, these expectations are achieved by following the provisions in the design codes which is based on experience, practice and judgment. However, this approach lacks systematic basis for evaluating the degree of conservativeness and may result inadequate or uneconomical designs. To assess the safety and to enforce the safety margins, it is essential to characterize and include all major sources of uncertainties associated with the analysis and design of structural systems.

Reliability based design methods have been proposed to include the effects of soil property variations in a more scientific way. Geotechnical problems are often dominated by uncertainty, such as inherent spatial variability of soil properties or scarcity of representative data. Engineers try to solve these problems using the traditional deterministic approach based on the safety factor, but this cannot explicitly deal with uncertainty, thus affecting the safety of engineering structures. In recent years reliability analyses and probabilistic methods have been applied in order to provide a more rational mathematical framework to incorporate different types of uncertainty into a geotechnical design.

Reliability based design methods could be used to address the geotechnical and structural strength requirements, such as the side friction and tip bearing capacity of foundations. Rather than calculating a deterministic factor of safety, a reliability based analysis will be more appropriate for geotechnical design if it is investigated in detail. Such analysis indicates the performance and reliability of a geotechnical problem, and can be used for risk-based decision making. To initiate a reliability analysis, random fields of soil properties are commonly generated to derive the required statistical parameters, e.g. mean and standard deviation. A method of reliability analysis is then selected for determining the probability of failure and the reliability index. Some commonly used techniques include First Order methods and Point Estimate method.

In fact, building codes are a bit vague on the issue of acceptable risk, partly because of the difficulty in assessing overall failure probabilities for systems as complex as entire buildings. Reliability calculations provide a means of evaluating the combined effects of uncertainties, and a means of distinguishing between conditions where uncertainties are particularly high or low. In spite of the fact that it has potential value, reliability theory has not been much used in routine geotechnical practice. There are two reasons for this. First, reliability theory involves terms and concepts that are not familiar to most geotechnical engineers. Second, it is commonly perceived that using reliability theory would require more data, time, and effort than are available in most circumstances.

### 1.2.Statement of the problem

Geotechnical engineers face challenges during design of foundations and other geotechnical applications. The challenges come from multiple sources of variability and uncertainty of design parameters. There are uncertainties in live load and soil properties such as; unit weight, cohesion & angle of friction. For many years, engineers have designed foundations and other geotechnical applications using deterministic methods. In deterministic methods, all uncertainties in the load and resistance are combined into a single factor of safety which, unfortunately, leads to uncertain safety margins in the design. The factor of safety used in conventional geotechnical practice is based on experience, practice and judgement. However, this approach lacks systematic basis for evaluating the degree of conservativeness and due to this it is common to use the same value of factor of safety for a given type of foundations, without regard to the degree of uncertainty involved in its calculation. The same value of safety factor is applied to conditions that involve widely varying degrees of uncertainty. Estimation of factor of safety in these ways does not reflect the inherent uncertainty in relation to bearing capacity parameters, rather leading to unreliable bearing capacity predictions. This will cause difficulties to design foundations with full confidence. If level of confidence is low during design, the geotechnical structures will be designed either below or above required standard. This may result inadequate or uneconomical designs i.e. will cause risks and economical losses. To assess the safety and to enforce the safety margins, it is essential to characterize and include all major sources of uncertainties associated with the analysis and design of structural systems. Therefore, to solve geotechnical problems confidentially proposing more rational alternative approaches for estimating bearing capacity of foundations is very important. Developing reliability concepts in simple ways, without more data, time, or effort than conventional geotechnical engineering practice solves geotechnical problems easily. This will play its own role for evaluation and reduction of uncertainties during design.

Therefore, analysis and design of shallow and deep foundations based on deterministic methods and reliability based design approaches is crucial to compare and contrast with safety and economic situations of geotechnical engineering problems.

### 1.3. Objective of the study

#### 1.3.1 General objective

The main objective of the study is to compare and contrast the stability and economic implication of foundation designs using the deterministic and reliability based design approaches.

#### 1.3.2 Specific objectives

The specific objectives of this study are:

- To determine engineering properties of the selected site foundation soils.
- To analyze typical shallow and deep foundation systems using the deterministic factor of safety approach and reliability based design.
- Comparison of safety margins and economic implications from both methods.
- To recommend the suitability of the two methods for the variety foundation options based on safety and economic evaluations..

## 1.4 Materials and methods

To address the objective of the research, the following methodologies of data collection and analysis are done.

Literatures related with the selected topic were assessed. Soil samples which will be suitable for spread and pile foundations were collected at Arada sub city high rise building sites. These samples were taken to laboratories to obtain engineering properties of soils such as friction angle and cohesion. Type of test which was conducted for this purpose was direct shear consolidated drained test.

Based on laboratory results bearing capacity calculation performed using both deterministic and reliability based design methods. Terzaghi, Meyerhof, Hansen and Vesic equations have been used. Reliability based design methods include First order second moment (FOSM), Second order second moment (SOSM), and Point estimation method (PEM) were conducted. Then reliability based design methods result are compared with deterministic methods. The most reasonable and logical bearing capacity results selected from both methods for design of foundations. After design of foundations economic analysis and factor of safety situations have been solved and the results are compared. Finally conclusion on findings and recommendations for further researches are done.

General flow chart of methods;

Step by step methodologies of the research summarized as follows;

Observation of previous similar studies related with selected topic

Collection of soil samples

Laboratory tests conducted to determine engineering properties of soils

Direct shear test conducted to determine shear strength parameters of soils.

Saturated and dry unit weight of collected soil samples determined by using available laboratory tests.

Soil properties determined based on parameters obtained in the laboratory such as cohesion and internal friction angle.

Bearing capacities calculated using the two methods i.e. Deterministic and reliability based design approaches.

Deterministic approaches which have been used in bearing capacity calculations are;

- Terzaghi method,
- Meyerhof method,
- Hansen method,
- Vesic method

Reliability based design methods are;

- First Order Second Moment (FOSM) method,
- Second Order Second Moment (SOSM) method, and Point Estimation Method (PEM) and

Bearing capacity values for collected soil types were also determined according to Ethiopian Standard with Euro Norms (ES EN) and EURO code.

Both approaches' bearing capacity results analyzed and compared.

Isolated spread footing and pile foundation designed depending upon selected bearing capacity results.

Safety conditions and economic implications of deterministic and reliability based design approaches compared.

Relevant findings summarized and recommendations ordered.

### 1.4. Organization of the thesis

A brief description of the work carried out under various chapters is discussed below.

Chapter 1 overviews about foundations, deterministic and reliability based design approaches. It also describes the aim , reasons and methodology of the thesis.

Chapter 2 mentions the related literatures which had been done previously. It explains different concepts with sub-titles. Types of sources of uncertainities are explained in this chapter with reference to different authors and practical judgements.

Chapter 3 deals with both deterministic and reliability design approaches. It expresses equations used under each categories.

Chapter 4 deals with the analysis and design of isolated spread footing using both approaches. Detail calculations of each method are discussed in this section. Several numerical, tabular and graphical bearing capacity results are also mentioned here. Finally comparision of spread footing design using both deterministic and reliability based design approaches is done by considering safety and economical conditions.

Chapter 5 discusses the conventional analysis and design method of pile foundation. This chapter also includes pile capacity analysis and design using load and resistance factor (LRFD) method for reliability based design case. At the end of the chapter results of the two approaches are summarized.

Chapter 6 concludes basic findings of the study. Important recommendations are also set in the final section of this chapter.

## 1.5.Scope of the study

The study focuses on analyses and design of shallow foundations specifically spread footing and pile foundation using deterministic safety factor and reliability based design approaches based on strength limit state I.

# CHAPTER TWO

# 2 LITERATURE REVIEW

## 2.1 Bearing capacity of foundations

The bearing capacity study of shallow foundations is a subject with a very long reference list. The first important contributions for bearing capacity equation are Prandtl (1921) and Reissner (1924), who considered a punch over a weightless semi-infinite space, and Sokolovski (1965), in regard to a ponderable soil, all under plane strain conditions.

The ultimate bearing capacity of strip footings is generally determined by the Terzaghi method (1943). Terzaghi's equation is an approximate solution which uses the superposition technique to combine the effects of cohesion c, soil weight γ and surcharge q. These contributions are expressed through three factors of bearing capacity Nc, Nγ & Nq. These bearing capacity factors are functions of angle of internal friction ϕ. Terzaghi (1943) used an approximate approach to the physical reality where only a global limit equilibrium of rigid blocks defined by the Prandtl failure mechanism was required, but considering the basal angle of the central wedge equal to ϕ, instead of $45^o + \phi/2$. Generally, Terzaghi's bearing capacity equation is;

$$q_u = k_1\, c\, N_c + q\, N_q + k_2\, \gamma' B\, N_\gamma \quad \text{…… Equation 2-1}$$

Where; qu is ultimate bearing capacity, k1 and k2 are factors for shape of foundation, c is cohesion, Nc, Nq and Nγ are bearing capacity factors, q is discharge, γ' is effective unit weight of the soil and B is breadth of the foundation.

Meyerhof (1951) obtained, with a similar technique of the Terzaghi's approach, approximate solutions to the plastic equilibrium of shallow foundations and deep foundations, assuming a different failure mechanism and like Terzaghi, expressing the results in the form of bearing capacity factors in terms of the angle of internal friction.

$$Q_{ult} = c\, N_c\, S_c\, d_c\, i_c + q\, N_q\, S_q\, d_q\, i_q + \tfrac{1}{2}\, \gamma\, B\, N_\gamma\, S_\gamma\, d_\gamma\, i_\gamma \quad \text{…… Equation 2-2}$$

Where; Sc,Sq and Sγ are shape factors, dc,dq and dγ are embedment depth factors and ic,iq and iγ are inclination factors of loads.

Bearing capacity factors;

$Nq = e^{(\tan\varphi \ X\ N\varphi)}$

$Nc = (Nq - 1)\cot\phi$

$N\gamma = (Nq - 1)\tan(1.4\phi)$ ….. Meyerhof

$N\gamma = 1.5\,(Nq - 1)\tan\phi$ ……. Hansen

$N\gamma = 2\,(Nq + 1)\tan\phi$ ……. Vesic

……………………………………………………………. Equation 2.3

In general, the majority of the authors coincide in the expressions employed to determine Nc & Nq, nevertheless, there is a great discrepancy with respect to the values of the factor Nγ. This is the principal reason which leads to variation in bearing capacity results.

Table 2-1 : Shape and depth factors

Shape factors

| *Factors* | *Meyerhof* | *Hansen* | *Vesic* |
|---|---|---|---|
| $s_c$ | $1 + 0.2\,N_\phi\left(\frac{B}{L}\right)$ | $1 + \frac{N_q}{N_c}\left(\frac{B}{L}\right)$ | |
| $s_q$ | $1 + 0.1\,N_\phi\left(\frac{B}{L}\right)$ for $\phi > 10°$ | $1 + \frac{B}{L}\tan\phi$ | The shape and depth factors of Vesic are the same as those of Hansen |
| $s_\gamma$ | $s_\gamma = s_q$ for $\phi > 10°$<br>$s_\gamma = s_q = 1$ for $\phi = 0$ | $1 - 0.4\,\frac{B}{L}$ | |
| $d_c$ | $1 + 0.2\,\sqrt{N_\phi}\left(\frac{D_f}{B}\right)$ | $1 + 0.4\left(\frac{D_f}{B}\right)$ | |
| $d_q$ | $1 + 0.1\,\sqrt{N_\phi}\left(\frac{D_f}{B}\right)$<br>for $\phi > 10°$ | $1 + 2\tan\phi\,(1 - \sin\phi)^2\left(\frac{D_f}{B}\right)$ | |
| $d_\gamma$ | $d_\gamma = d_q$ for $\phi > 10°$<br>$d_\gamma = d_q = 1$ for $\phi = 0$ | 1 for all $\phi$<br>*Note:* Vesic's $s$ and $d$ factors = Hansen's $s$ and $d$ factors | The shape and depth factors of Vesic are the same as those of Hansen |

Table 2-2 : Inclination factors

Inclination factors

| *Factors* | *Meyerhof* | *Hansen* | *Vesic* |
|---|---|---|---|
| $i_c$ | $\left(1-\frac{\alpha^\circ}{90}\right)^2$ for any $\phi$ | $i_q - \frac{1-i_q}{N_q-1}$ for $\phi > 0$ | Same as Hansen for $\phi > 0$ |
| | | $0.5\left(1-\frac{Q_h}{A_f c_a}\right)^{\frac{1}{2}}$ for $\phi = 0$ | $1-\frac{mQ_h}{A_f c_a N_c}$ |
| $i_q$ | $i_q = i_c$ for any $\phi$ | $\left(1-\frac{0.5Q_h}{Q_u + A_f c_a \cot\phi}\right)^5$ | $\left(1-\frac{Q_h}{Q_u + A_f c_a \cot\phi}\right)^m$ |
| $i_\gamma$ | $\left(1-\frac{\alpha^\circ}{\phi^\circ}\right)^2$ for $\phi > 0$ <br> $i_\gamma = 0$ for $\phi = 0$ | $\left(1-\frac{0.7Q_h}{Q_u + A_f c_a \cot\phi}\right)^5$ | $\left(1-\frac{Q_h}{Q_u + A_f c_a \cot\phi}\right)^{m+1}$ |

$m = m_B = \frac{2+B/L}{1+B/L}$ with $Q_h$ parallel to $B$ $\qquad m = m_L = \frac{2+B/L}{1+B/L}$ with $Q_h$ parallel to $L$

Terzaghi equation is the most popular used equation by engineers in practice and by researchers. For example, a research has been done by Felipe Alberto in 2000 where he tested Terzaghi, Hansen, Meyerhof equations experimentally. He used circular plate loading testing method and compared the results of bearing capacity and bearing capacity factors. He found out that Terzaghi's equation produced very close values to the actual ones and and he mentioned this method is the safest equation compared to the other ones.

## 2.2 Uncertainty in geotechnical engineering

For assessment of reliability, Morgenstern (1997) classifies the uncertainties into three categories i.e. i) parameter uncertainty; ii) model uncertainty; and iii) human uncertainty. In engineering design, the influence and significance of the first two modes of uncertainty are important, while the third mode of uncertainty falls within the domain of regulatory agencies. While parameter uncertainty is easier to handle, the model uncertainty can be handled if the mechanism of failure is known. It is generally agreed that the variability associated with geotechnical properties should be divided in to three main sources, viz., inherent variability, measurement uncertainty, and transformation uncertainty (Vanmarcke 1977a; Baecher 1982; Tang 1984; Phoon and Kulhawy 1999a).

### 2.2.1 Inherent variability

Inherent variability of a soil parameter is attributed to the natural geological processes, which are responsible for depositional behavior and stress history of soil under consideration. The fluctuations of soil property about the mean can be modeled using a zero-mean stationary random field (Vanmarcke 1977a; Phoon et al. 2003a). A detailed list of the fluctuations in terms of coefficients of variation for some of the laboratory and in-situ soil parameters, along with the respective scales of fluctuation in horizontal and vertical directions are presented by Kulhawy (1992), Lacasse and Nadim (1996), Duncan (2000).

### 2.2.2 Measurement uncertainty

Measurement uncertainty is described in terms of accuracy and is affected by bias (systematic error) and precision (random error). It arises mainly from three sources; equipment errors, procedural-operator errors, and random testing effects, and can be evaluated from data provided by the manufacturer, operator responsible for laboratory tests and/or scaled tests. Nonetheless the recommendations from regulatory authorities regarding the quality of produced data, the measuring equipment and other devices responsible for the measurement of in-situ or laboratory soil properties often show variations in its geometry, however small it may be. There may be many limitations in the formulation of guidelines for testing, and the understanding and implementation of these guidelines vary from operator to operator and contribute to procedural-operator errors in the measurement. The third factor, which contributes to the measurement uncertainty, random testing error, refers to the remaining scatter in the test results that is not assignable to specific testing parameters and is not caused by inherent soil variability (Jaksa et al. 1997; Phoon and Kulhawy 1999a).

### 2.2.3 Transformation uncertainty

Computation models, especially in the geotechnical field contain considerable uncertainties due to various reasons, e.g. simplification of the equilibrium or deformation analysis, ignoring 3-D effects etc. (JCSS 2000; Phoon and Kulhawy 2003, Zhang et al. 2004). Expected mean values and standard deviations of these factors may be assessed on the basis of empirical or experimental data, on comparison with more advanced computation models. Many design parameters used in geotechnical engineering are obtained from in-situ and laboratory test results. To account for this uncertainty, the model or transformation uncertainty parameter is used.

## 2.3 Risk associated with evaluation of bearing capacity

Researchers have shown that bearing capacity failure happens due to shear failure of the soil beneath the footing. They have observed three predominant failure types. The first one is the general shear failure, the second one the local shear failure and the third one is the punching shear failure. The general shear failure happens in dense sand of Dr (relative density of sand) > 70% and in saturated normally consolidated clays (Coduto, 2001). This type of failure is sudden and happens when the settlement reaches 7 % of the foundation width (Coduto, 2001). When this type of failure happens a clear bulge appears on the ground surface near the foundation. This is the most common type of failure. The second type is the local shear failure which happens in medium dense sand that has a relative density between 70% and 35% (Coduto, 2001). This type of failure is not sudden and happens when the settlement exceeds 8% of the foundation width. The failure surface will gradually extend outward from the foundation but a sudden failure may not ever happen and the foundation will continue to sink into the soil (Coduto, 2001).

The third kind of failure is the punching shear failure. This type of failure happens in loose sands of relative density of less than 35%. In this type of failure the settlement will be between 15% - 20% of the foundation width. Bulging may never happen and the failure surface which is a vertical and follows the perimeter of the foundation and it will never reach the ground surface.

Numerous studies have been made in the recent past on the importance of implementation of risk and reliability concepts and extending its potential benefits in the field of geotechnical engineering. The Construction Industry Research and Information Association (CIRIA 2001) defines the risk as “the probability (or likelihood) of an unwanted uncertain event, and its unwanted consequences for objectives”. These factors when used cautiously, guarantee to a reasonable degree of confidence in the safety and serviceability of foundation system. The success or failure of the site investigation and the structure depends on the approach to risk (CIRIA 2001). Risk management is not new. Traditionally it has been in use with risk remaining implicit accounting for various sources and levels of uncertainty in footing loads and soil resistance and engineering design methods and processes have historically managed by experience and subjective judgment (Paikowsky 2002). Terzaghi (1936) observed that no geotechnical site is truly homogeneous in engineering properties. However, in general, to simplify the analysis, analytical and transformation models are used to interpret results of site investigation using simplified assumptions and approximations. But, in reality due to the complexity in soil formation and depositional processes, soil behavior is seldom homogenous.

## 2.4 Reliability based design methods in geotechnical engineering

Christian (2004) outlined the following additional approaches for dealing with geotechnical uncertainties:

- Ignoring it
- Being conservative
- Using the observational method
- Quantifying it

Ignoring the uncertainties lead to baseless decisions with catastrophic consequences while being conservative, although guarantees safety, is usually uneconomical. The two design approaches do not meet the fundamental design requirements of simultaneously achieving safety and economy. The observational method or "learn-as you go" is suitable for large projects with complex ground conditions necessitating contract documents tailored for the specific project. However, under the normal design setting where a complete design is required to facilitate tendering of the construction stage of the project by various contractors, the approach is not feasible. Concerning quantifying the uncertainties, there is consensus that the approach is the most rational and transparent. Christian (2004) asserts that quantifying the uncertainties is consistent with the philosophy of the observational method and should therefore be considered as a logical extension of the approach that accommodates modern developments in probabilistic methods.

Communication of risk within a transparent and rational framework is further motivated by the increasing pressure in code harmonisation as results of greater economic cooperation and integration brought by the advent of the World Trade Organisation, public involvement in defining acceptable risk levels, and risk-sharing among client, consultant, insurer and financier. The need for a framework that can treat uncertainties in transparent and rational manner can not be over emphasised. The critical question now is what framework is capable of dealing with uncertainties in this desired manner?

Historically, probability theory has been the primary tool for modelling uncertainties. Therefore the framework should ideally be based on probability theory. If the framework is not reliability analysis, then what alternative is available (Phoon, et al, 2003; Phoon, 2004)? Certainly with the current state of knowledge, only reliability analysis and design can provide a consistent method for propagating uncertainties throughout the design process. In addition to dealing with uncertainties, the reliability based design framework provides a unifying framework for risk assessment across disciplines and national boundaries. This is important for achieving compatibility between structural design and geotechnical design so as to avoid the current scenario whereby different approaches are applied to two sides of the soil-structure interface.

Reliability analysis can be carried out thorough study of the related uncertainties. In regarding to the bearing capacity of foundations, there are many uncertainties involved, e.g. variation of soil properties, variation of magnitude and direction of loading, uncertainties in the bearing capacity equations, distribution and correlation of the uncertainties. Among soil properties; unit weight, cohesion and friction angle are the most frequently studied variables regarding to the reliability analysis of bearing capacity of foundations. As these three parameters are most directly used to evaluate bearing capacity of foundations in many available methods. The variation of a parameter is described with the coefficient of variation (COV) of its distribution. Research found that, unit weight varies in a relatively limited range with COV between 1- 10%. COV values for friction angle are in a range of 5-20% for sands and 7-56% for clays. The most highly varied and hardest to estimated parameter is the COV of shear strength of clays especially that of undrained shear strength. For saturated clays, an increase of 1% of water content in saturated clay may cause a reduction of 20% of the soil's undrained shear strength "Muni (2000) ". In unsaturated soils, due to the appearance of suction, a decrease of water content will result in the increase of apparent cohesion in soils "Fredlund et al. (1978)".

Foundation is designed to resist against loading from upper structures. The variation of loading needs to be considered in the reliability analysis of foundations. The variation of loads can be narrow or wide, depending on the nature of the loads. The first and foremost phase in the formulation of probabilistic techniques in geotechnical engineering is that of having the information with regard to the subsoil conditions and its variations, at least within the zone of interest, geotechnical properties, ground water table, suction characteristics in case if the soil is unsaturated, etc. Variations are expressed in terms of mean or average values and the coefficients of variation defined in terms of the ratio of standard deviation and mean value expressed as percentage. A successful geotechnical design depends largely on how best the designer selects the basic soil parameters of the site under consideration from in-situ and/or laboratory test results. These values are subjective, and depend on the individual decisions based on personal experience and judgment of the engineer in-charge. Surprisingly, the higher variability with which the predictions have been done against the measured performance of foundations and embankments reveals that there is little consensus among the designers on the values of soil parameters considered in the analysis (Kay 1993).

Well known probabilistic techniques include First Order Second Moment (FOSM), Second Order Second Moment (SOSM) and the Point Estimate method, for dealing with uncertainties and for implementing probabilistic concepts into geotechnical analyses in a more rational way. Actually, assuming soil parameters, such as the friction angle and the cohesion, as random variables described by a certain probability density function, then probabilistic methods are applied to assess the probability density function and the statistical values of a limit state function (e.g. the bearing capacity) which depends on the input variables. Thus, failure probabilities and reliability indexes can be estimated from the output results, leading to a more meaningful evaluation of safety design (Consolata Russelli Stuttgart, April 2008).

## 2.5 Reliability index of bearing capacity

Reliability index of a geotechnical structure is a measure of the safety that takes into account the inherent uncertainties of the input variables. A widely used reliability index is the Hasofer and Lind (1974) index defined as the shortest distance from the mean value point of the random variables to the limit state surface in units of directional standard deviations. Reliability index mean a lower probability of failure and a safer overall structure (Ali Alhajami, University of Nebraska-Lincoln).

The first order reliability method is based on first-order Taylor series expansion approximation. The performance function is linearized at mean values of the random variables. A measure of reliability can be estimated by introducing the reliability index β that is based on the mean and standard deviation of Z as (Cornell).

Conventional deterministic analyses may not always capture the soil variability and the uncertainty in soil properties. In these cases, a reliability analyses may be more appropriate as statistical properties can be used to determine the probability of failure and reliability index. The importance of reliability analyses has been recognized by the recently implemented Euro code 7 and is encouraged to use whenever suitable. A recommended deterministic solution by Euro code 7 has been implemented while characteristic values were input as random variables for cohesion and friction angle. Many studies indicated that, the reliability indexes exhibited huge difference using different methods when limit state function is not normally distributed.

The probability of failure (pf) can be estimated from the reliability index β using the established equation $Pf = 1 - \Phi(\beta) = \Phi(-\beta)$, where Φ is the cumulative distribution (CDF) of the standard normal variate. The relationship is exact when the limit state surface is planar and the parameters follow normal distributions, and approximate otherwise. Formula $pf = \Phi(-\beta)$ expresses the relationship between failure probability (pf) and reliability index (β). The acceptable βT is in essence the maximum acceptable failure probability. For example, determining acceptable βT = 3.0 means the acceptable maximum failure probability is 0.001.

The utilization of the LRFD method requires the selection of a set of target reliability levels which determines the probability of failure and hence the magnitude of the load and resistance factors. The probability of failure represents the probability for the condition in which the resistance multiplied by the resistance factors will be less than the load multiplied by the load factors. When fitting LRFD to ASD, the issue is less significant as practically the factors are established to conform (often conservatively) to existing factors of safety. When calibrating for a database, however, the establishment of an acceptable probability of failure is cardinal, including the question of new design versus existing state of practice. An approximate relationship between probability of failure and target reliability for a lognormal distribution was presented by Rosenbleuth and Estava (1972):

$p_f = 460e^{-4.3\beta}$ …………………………………………………………………………………….. Equation 2-3

and is commonly in use; e.g. Withiam (1998). Baecher (2001) shows, however, that this approximation is not so accurate below of β about 2.5.

Barker et al(1991) reduced the target reliability index for driven piles to a value between 2.0 and 2.5, especially for a group system effect. Paikowsky (2004) suggested an initial target reliability index between 2.0 and 2.5 for a pile group, and 3.0 for a single pile. Paikowsky (2004) also recommended target reliability indices of 2.33 (corresponding to 1% probability of failure) and 3.00 (corresponding to 0.1% probability of failure) for representing redundant and non-redundant pile groups, respectively. As suggested by Barker et al(1991) and Paikowsky (2004), five levels (2.0, 2.5, 3.0, 3.5 and 4.0) of target reliability index will be considered and the corresponding resistance factors calculated in more studies. The authors recommended that, engineers should be careful in using the reliability index, especially when performance function is highly nonlinear and random variables are not normal distribution.

## 2.6 Load and resistance factor design method

Load and resistance factor design (LRFD) is conceptually a more advanced design method than the allowable stress design (ASD). The key improvements of LRFD over the traditional ASD are the ability to provide a more consistent level of reliability and the possibility of accounting for load and resistance uncertainties separately (Foye et al2006).

Researches have been done to study the influence of variation of soil properties on bearing capacity of shallow foundations "Cherubini (2000); Honjo et al. (2000); Phoon et al. (2003); Alawneh et al. (2006) ". Though load is one of the most variable parameters in shallow foundation design, not much discussion about the effect of variation of inclined load on the bearing capacity of foundations was available in publications. "Honjo et al. (2000)" used First Order Reliability Analysis (FORM) to study the variation of inclination factor on the reliability of shallow foundations with a modified Terzaghi equation. Load and resistance factor design (LRFD) method was introduced into shallow and deep foundation design "Paikowsky et al. (2004); Paikowsky et al. (2010) ". Orr (2000) discussed the selection of partial factors and suggested that engineers should be careful in selecting these factors in terms of favorable or unfavorable actions. In many cases of shallow foundation design, either horizontal or vertical loads can be unfavorable. Applying same partial factors to these two actions might not be proper, as the variations of the two actions are not the same in many cases.

## 2.7 Factor of safety

In the recent past, in many regulations and codes, the safety of geotechnical works has been entrusted to an approach called ASD (Allowable Stress Design) based on the so called global safety factor expressed as the ratio between the resistances and the actions expressed both in terms of forces, moments or stresses considering obviously all the possible kinematics in the examined case. Normally this ratio was based, even if not appositely specified, on actions and resistances calculated through the mean values of the relating input parameters

Complication is represented by the circumstance that in the classical definition of the Safety Factor intended as ratio some forces, moments could be considered either as additional contribute to Resistance or as decrement to the Action with numerically different results (De Mello 1989, Li et al., 1993, Rethati, 1988).

Consequently codes and regulations have been developed for which the resistances are evaluated through characteristic values (even on statistical basis) of the input parameters penalized with appropriate “partial safety factors”, while the Actions are analogously amplified. Operating in this way it is possible to take into account in a certain way of the variability of the parameters of resistance penalizing them and at the same time amplifying the actions, arriving at a final State limit approach (ULS).

A more comprehensive and complex approach in the evaluation of safety is given by probabilistic methods, that are based on the evaluation of the reliability through the so called reliability index β through which it is possible to reach the knowledgeof the probability connected to the unfavorable event considered.

Harr (1987) defined reliability as: “Reliability is the probability of success “(of a structure on soil or rock). A more precise definition is the probabilistic assessment of the likelihood of the adequate performance of a system for a specified period of time under proposed operating conditions. The acceptance of a Reliability level must be viewed within the context of possible costs, risks and associated social benefits.

# CHAPTER THREE

# 3 BEARING CAPACITY ANALYSIS METHODS

## 3.1. Deterministic Analysis Methods

### 3.1.1 Terzaghi's bearing capacity theory

Terzaghi developed bearing capacity calculation method for general shear failure case in 1943. His equation takes considerations of soil cohesion, soil friction, embedment, surcharge and self-weight. He didn’t consider inclination effects in his equation. Terzaghi's bearing capacity equation is:

$$q_u = k_1 c N_c + q N_q + k_2 \gamma' B N_\gamma \quad \text{Equation 3-1}$$

Where; qu is ultimate bearing capacity, k1 and k2 are factors for shape of foundation, c is cohesion, Nc, Nq and Nγ are bearing capacity factors, q is discharge, γ' is effective unit weight of the soil when saturated or total unit weight when not fully saturated and B is breadth of the foundation.

**Assumptions in Terzaghi's Bearing Capacity Theory**

- Depth of foundation is less than or equal to its width.
- Base of the footing is rough.
- Soil above bottom of foundation has no shear strength; is only a surcharge load against the overturning load.
- Surcharge up to the base of footing is considered.
- Load applied is vertical and non-eccentric.
- The soil is homogenous and isotropic.
- Footing length to width (L/B) ratio is infinite.

### 3.1.2 Meyerhof's bearing capacity theory

The form of equation used by Meyerhof (1951) for determining ultimate bearing capacity of symmetrically loaded strip footings is the same as that of Terzaghi but his approach to solve the problem is different. He assumed that the logarithmic failure surface ends at the ground surface, and as such took into account the resistance offered by the soil and surface of the footing above the base level of the foundation. Meyerhof included shape, depth and inclination factors in his equation.

$$q_{ult} = c\,N_c\,S_c\,d_c\,i_c + q\,N_q\,S_q\,d_q\,i_q + \tfrac{1}{2}\,\gamma\,B\,N_\gamma\,S_\gamma\,d_\gamma\,i_\gamma \quad \text{.................................... Equation 3-2}$$

Where; $S_c$, $S_q$ and $S_\gamma$ are shape factors, $d_c$, $d_q$ and $d_\gamma$ are embedment depth factors and $i_c$, $i_q$ and $i_\gamma$ are inclination factors of loads.

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = (N_q - 1)\tan(1.4\,\phi)$$

$$N_q = e^{\Pi \tan\phi}\,tan^2(45 + \phi/2)$$

.................................................................. Equation 3-3

### 3.1.3 Hansen's bearing capacity theory

Hansen (1961) developed the ultimate bearing capacity equation with some modification from previous equations. According to Hansen, the ultimate bearing capacity is given by:

$$q_{ult} = c\,N_c\,S_c\,d_c\,i_c + q\,N_q\,S_q\,d_q\,i_q + \tfrac{1}{2}\,\gamma\,B\,N_\gamma\,S_\gamma\,d_\gamma\,i_\gamma \quad \text{................................ Equation 3-4}$$

Where; $S_c$, $S_q$ and $S_\gamma$ are shape factors, $d_c$, $d_q$ and $d_\gamma$ are embedment depth factors and $i_c$, $i_q$ and $i_\gamma$ are inclination factors of loads.

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = 1.5\,(N_q - 1)\tan\phi$$

$$N_q = e^{\Pi.\tan\varphi}\,tan^2(45 + \phi/2)$$

................................................................. Equation 3-5

### 3.1.4 Vesic's bearing capacity theory

Vesic (1973) confirmed that the basic nature of failure surfaces in soil as suggested by Terzaghi as incorrect. However, the angle which the inclined surfaces make with the horizontal was found to be closer to $45^o + \phi/2$ instead of $\phi$. According to Vesic, the ultimate bearing capacity is given by:

$$q_{ult} = c\,N_c\,S_c\,d_c\,i_c + q\,N_q\,S_q\,d_q\,i_q + \tfrac{1}{2}\,\gamma\,B\,N_\gamma\,S_\gamma\,d_\gamma\,i_\gamma \quad \text{....................... Equation 3-6}$$

Where; Sc,Sq and Sγ are shape factors, dc,dq and dγ are embedment depth factors and ic,iq and iγ are inclination factors of loads.

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = 2(N_q + 1)\tan\phi$$

$$N_q = e^{\Pi \tan\varphi}\, tan^2(45 + \phi/2)$$

……………………………………………………………. Equation 3-7

### 3.1.5 Ethiopian Standars with Euro Norms ( ES EN) 1997-2015 bearing capacity calculation

Bearing capacity according to ES EN 1997-2015 is calculated as;

$R/A = c\, N_c\, S_c\, b_c\, i_c + q\, N_q\, S_q\, b_q\, i_q + \frac{1}{2}\, \gamma\, B\, N_\gamma\, S_\gamma\, b_\gamma\, i_\gamma$ ………………………………… Equation 3-8

Where; Sc,Sq and Sγ are shape factors bc,bq and bγ are dimension factors and ic,iq and iγ are inclination factors of loads.

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = 2(N_q - 1)\tan\phi$$

$$N_q = e^{\Pi \tan\varphi}\, tan^2(45 + \phi/2)$$

……………………………………………………………. Equation 3-9

## 3.2. Reliability based analysis methods

### 3.2.1 The First Order Second Moment (FOSM) method

*3.2.1.1 Definition and methodology*

FOSM produces a linearization around the mean value of the input random variables of a probabilistic problem. This method uses a Taylor's series expansion of the performance function to be evaluated to determine the values of its first two central moments, mean value and standard deviation, depending on the input variables. This expansion is truncated after the linear term and, for this reason, the method is called "First Order". It is called a second moment method because the variance is a form of the second moment and is the highest order statistical result used in the analysis. If the number of uncertain variables is N, this method requires either evaluating N partial derivatives of the performance function or performing a numerical approximation using evaluations at 2N+1 points. Considering a performance function Z of N random variables Xi, as for example the bearing capacity as function of the cohesion and the friction angle, its Taylor's series expansion about the mean value of the random variables μX1,…,μXn, truncated after first order terms, gives

$$Z(X1,\dots,Xn) = Z(\mu x_1,\dots,\mu x_n) + \sum_{i=1}^{n}(Xi - \mu xi).\ \frac{\partial Z}{\partial xi} \quad \text{.................................... Equation 3-10}$$

The derivatives are evaluated at μX1… μXn, considered as linearization points. The mean value and the variance of the performance function are given approximately by the following equations.

$$\mu\, z(x1,\dots,xn) \approx Z(\mu x_1,\dots,\mu x_n) \quad \text{............................................................. Equation 3-11}$$

$$Var(Z(x1,\dots,xn)) \approx \sum_{i=1}^{n}\left(\frac{\partial Z}{\partial xi}\right)^2 .\, Var(Xi) + 2\sum_{i=1}^{n}\sum_{i=1}^{n}\left(\frac{\partial Z}{\partial xi}.\frac{\partial Z}{\partial xj}\right) Cov(Xi,\ Xj)$$

$$\text{................................................ Equation 3-12}$$

If the random variables are uncorrelated, then the term with the covariance drops out. In general, if the number of uncertain variables is n, this method requires either evaluating 'n' partial derivatives of the performance function or performing a numerical approximation using evaluations at 2n+1 point.

*3.2.1.2 Advantages and limitations of the FOSM method*

The most important advantages of the FOSM method include the following:

• The FOSM method is exact for linear performance functions.

• The summation terms of Equations 3-12 and 3-14 provide an explicit indication of the relative contribution of uncertainty of each random variable.

To avoid the misuse of FOSM method in probabilistic analyses, one should be referred of its limitations, which are listed below:

• Due to the Taylor's series truncation after first order terms, the accuracy of the method deteriorates if second and higher derivatives of the performance function are significant. Thus the accuracy of the FOSM method diminishes as the non-linearity of a function increases.

Unfortunately, retaining second and higher order terms of the Taylor's series expansion of a complex function with more than one input variable are mathematically complex.

• The skewness of the output probability density function is not provided.

• As the level of uncertainty in the input variables increases and their probability density functions become more skewed the accuracy of the FOSM method decreases.

• Additional assumptions on the output probability density function must be made to estimate any probability of failure; moreover the reliability index is not uniquely defined, because it depends on the safety format considered (e.g. R-L=0, where R = resistance and L = load).

• The shape of the probability density function of the input variables is not taken into account; the random variables are described using only their mean and standard deviation. In this way no information about the shape of the probability density function of the output is provided, but it has to be assumed. This assumption introduces a source of inaccuracy.

• Finally, the FOSM method is applied primarily to problems without spatial correlation among input variables. With extra calculation effort, the method can be applied for two correlated random variables, but this can be very cumbersome.

### 3.2.2 The Second Order Second Moment (SOSM) method

The Second Order Second Moment method (SOSM) represents a slight extension of FOSM method. Actually with SOSM method it is possible to include second order terms of the Taylor's series expansion in the evaluation of the mean value of a performance function.

Considering the variances, or standard deviations, of two random variables X, Y as known, the mean value of Z(X, Y) is given by where all derivatives are evaluated at the mean value of the input variables. The term including the covariance drops out if the variables are uncorrelated.

$$\mu z(x, y) \approx Z(\mu x, \mu y) + (x - \mu x)\frac{\partial Z}{\partial x} + (y - \mu y)\frac{\partial Z}{\partial y} + \frac{1}{2}.\mathrm{Var}(X)\frac{\partial^2 Z}{\partial x^2} + \frac{1}{2}.\mathrm{Var}(Y)\frac{\partial^2 Z}{\partial y^2}$$

$$+ \mathrm{Cov}(X, Y).\frac{\partial^2 Z}{\partial x . \partial y} \quad \text{………………………………} \quad \text{Equation 3-13}$$

When compared with FOSM method, the obvious advantage of SOSM method is that the calculated mean value is more accurate because second order terms are considered in the analysis.

### 3.2.3 The Two Point Estimate Method (PEM)

#### *3.2.3.1 Definition*

Another alternative method to evaluate statistical moments of a performance function is the Point Estimate Method, or shortly PEM. The Point Estimate method was first developed by ROSENBLUETH (1975). The Two Point Estimate method is a computationally straightforward technique for uncertainty analysis, capable of estimating statistical moments of a model output involving several stochastic variables, correlated or uncorrelated, symmetric or asymmetric. It is fundamentally a weighted average method similar to numerical integration formulas involving sampling points and weighting parameters.

The basic idea of this method is to replace the probability distributions of continuous random variables by discrete equivalent distributions having the same first three central moments, to calculate then the mean value, standard deviation and skewness of a performance function, which depends on the input variables.

To do this, two point estimates are considered at one standard deviation on either side of the mean value from each distribution representing the random variables. Then the performance function is calculated for every possible combination of the point estimates, producing 2n solutions, where n is the number of the random variables involved. Then the mean value, standard deviation and skewness of the performance function can be found from these 2n solutions.

PEM does not provide a full distribution of the output variable, it requires little knowledge of probability concepts and could be applied for any probability distribution. The procedure for implementing the PEM is clearly described in the next section.

#### *3.2.3.2 The procedure for implementing the PEM*

The procedure for implementing the PEM is described below step by step.

1. First of all a performance function Z (Xi) depending on n random variables Xi should be considered.

2. Then the locations of the sampling points for every random variable should be estimated. To do this one should first evaluate the so-called standard deviation units ξxi+ and ξxi-, which depend on the skewness coefficient νxi of the input variables and given by;

$$\xi xi+ = \frac{\nu xi}{2} + (1 + \left(\frac{\nu xi}{2}\right)^2)^{1/2}$$

$$\xi xi- = \xi xi + - \nu xi$$

…………………………………………………………………………………… Equation 3-14

If the input variables are symmetrically distributed, the standard deviation units will be both equal to unity. Knowing the mean value μxi and the standard deviation σxi of the input variables, the corresponding sampling point locations xi- and xi+ can be calculated using the following formulae:

$$xi+ = \mu xi + \xi xi + . \sigma xi$$

$$xi- = \mu xi - \xi xi -. \sigma xi$$

…………………………………………………………………………………… Equation 3-15

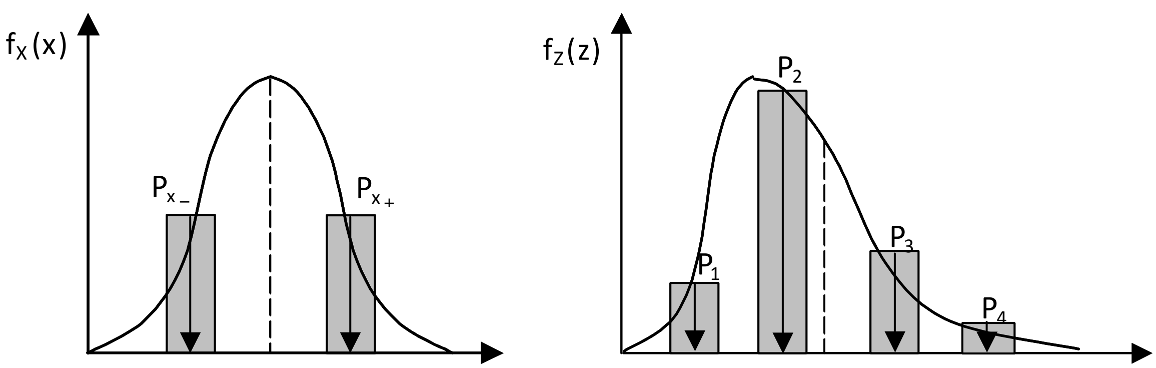



Figure 3-1 : Sampling point locations and weights for a single random variable (a) and for a function depending on two random variables (b)

In Fig. 3-1(a) and 3-1(b) the sampling point locations for a single random variable and for a function Z depending on two random variables X and Y are shown.

3. The weights Pi, also called probability concentrations, can now be determined to obtain all the point estimates. As a probability density function encloses an area of unity, then the weights must also sum to unity and they have to be positive. The weights of the random

variables are given by different expressions depending on the number of the input variables and on their correlation. For a single random variable (ROSENBLUETH, 1975) the weights are easily calculated using the standard deviation units as;

$$Pxi+ = \frac{\xi xi +}{(\xi xi + \; + \xi xi-)}$$

$$Pxi - = 1 - Pxi +$$

………………………………..……………… Equation 3-16

When the random variable is symmetric then the weights are both equal to 0.5. For two correlated random variables (ROSENBLUETH, 1981) the weights are given as follows;

$Ps1s2 = Pxs1. Pxs2 + s1.s2\ (\rho x1x2 / \left((1 + \frac{\nu xi}{2}\right)^3). \left((1 + \frac{\nu xi}{2}\right)^3))^{1/2})$ ……………… Equation 3-17

Where Ps1s2 is the associated weights, with Pxs1 and Pxs2 being the weights for the input variables evaluated as single variables. $\rho x1x2$ is the correlation coefficient between the variables x1 and x2, s1 and s2 take positive sign for points greater than the mean value of the variables and negative sign for points smaller than the mean value.

The sign product s1s2 determines the sign of the correlation coefficient and the subscripts of the weight P indicate the location of the point that is being weighted. For example, considering the point evaluated at $(x1+, x1-)$ = $(\mu xi + (\xi xi+) . \sigma xi, \mu xi - (\xi xi-) . \sigma xi \quad)$ then s1 = + and s2 = - , resulting in a negative with a weight denoted by P+-.

When the variables are uncorrelated then $\rho x1x2$ will be zero. First of all, if the skewness coefficient of the input variables has different sign then the radicand under the square root can be negative, which is mathematically impossible. This can happen for example if one input variable has a negatively skewed distribution and the other a symmetrically or positively skewed distribution.

Secondly if the skewness coefficient of the input variables is equal to –2 then the denominator of the second term of Equation 3-19 tends to infinity, giving then infinite weights. Moreover this formula can sometimes give negative values. This fact is unacceptable, because the weights are described as probability values, which are always positive by definition. For example, negative values of the weights can occur when the random variables are symmetric and perfectly correlated (i.e. $\rho x1x2 = \pm 1$). Considering all the drawbacks of Equation 3-14, it is necessary to establish some conditions for its use, i.e.

1. $Ps1s2 > Pxs1. Pxs2 + s1.s2\ (\rho x1x2/\left((1+\left(\frac{\nu x1}{2}\right)^{3)}\right).\left((1+\left(\frac{\nu x2}{2}\right)^{3)}\right))^{1/2}\ )$ ………. Equation 3-18

2. $\left((1+\left(\frac{\nu x1}{2}\right)^{3}\right).\left((1+\left(\frac{\nu x2}{2}\right)^{3}\right) > 0$ ……………………………………………………………. Equation 3-19

3. $\nu x1 \neq -2$ and/or $\nu x2 \neq -2$ ……………………………….……………. Equation 3-20

The first condition assures that the weights are positive. The third condition is actually already implied in the second one.

To overcome the problem in ROSENBLUETH's formula, a better definition for two correlated random variables, being symmetrically distributed, is given by CHRISTIAN et al. (1999), where the weights can be evaluated as;

$P_{+-} = P_{-+} = (\ P_{x1} \pm).(\ P_{x2\ -/+).(1\ -}\ \rho x1x2)$

$P_{++} = P_{--} = (\ P_{x1} \pm).(\ P_{x2} \pm_{).(1\ -}\ \rho x1x2)$

……………………………………………………………………. Equation 3-18

For n symmetrically distributed and correlated random variables, CHRISTIAN et al. (1999) define the weight as;

$P_{s1,\,s2,\ldots,sn} = \left(\frac{1}{2^n}\right) \cdot (1 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (si.\, sj.\, \rho ij))$ ………………………………………. Equation 3-19

4. Now it is possible to determine the performance function $Z(x_i)$ at each sampling point located at xi+ and xi-.

Finally the first three moments of the performance function, respectively the mean value, the variance and the skewness coefficient, can be determined using the following equations.

$\mu_{Z(xi)=} \sum_{i=1}^{2^n} (Pi\ .\ Z(Xi = xi)$ ……………………………………………………….…..…………. Equation 3-20

$$\sigma^2 z(xi) = \sum_{i=1}^{2^n} (Pi \,.\, Z(Xi = xi) - \mu z(xi))^2 \quad \text{.......... Equation 3-24}$$

$$\nu\, z(xi) = \frac{1}{\sigma^3 z(xi)} \sum_{i=1}^{2^n} (Pi \,.\, Z(Xi = xi) - \mu z(xi))^3 \quad \text{.......... Equation 3-21}$$

the standard deviation of the performance function is easily obtained as;

$$\sigma z(xi) = \left( \sum_{i=1}^{2^n} (Pi \,.\, Z(Xi = xi) - \mu z(xi))^2 \right)^{1/2} \quad \text{.......... Equation 3-22}$$

*3.2.3.3 Advantages and limitations of the PEM*

The most important advantages of PEM in comparison to FOSM and SOSM methods are:

• As with the FOSM and SOSM methods, the PEM does not require the knowledge of the particular shape of the probability density function of the input random variables. Moreover the PEM furnishes the exact solution for linear performance functions.

• It provides not only the mean value and the standard deviation,but also the skewness coefficient of a performance function, giving then more accurate results than FOSM, SOSM methods, with little or no increase in computational effort.

• The PEM may better capture the behaviour of non-linear functions.

• To evaluate the statistical values of a performance function there is no need to compute the derivatives, nor even their continuity let alone their existence.

• As a non-iterative procedure, the PEM overcomes the convergence problems with less time consuming.

• It can be also applied to problems with spatial correlation among multiple input variables, even if more computational effort is required.

When compared with Monte Carlo method, the PEM results in terms of mean value and standard deviation are in good agreement with those of MCS, with smaller computational effort for a comparable degree of accuracy.

Limitations of the PEM are:

- If more accuracy is required, then a larger number of input variables is necessary and higher moments have to be considered,thus increasing the number of calculations.

- Results are poor and less accurate for discontinuous functions or functions having discontinuous first derivatives (CHRISTIAN and BÄCHER, 1999).

## 3.3 Reliability Index (β) and Probability of Failure(pf)

A simple method of expressing the probability of failure is to use the "reliability index," β**.** In the context of reliability analysis, failure is defined as the conditions where a predefined limit state is reached. Load and resistance factors are selected to insure that each possible limit state has an acceptably small probability of occurrence. The probability of failure can be determined if the mean and standard deviation of the resistance and load are known. A quantitative measure of safety is the probability of survival, ps, given by:

$$ps = P(R > Q) \quad \text{Equation 3-23}$$

The complement of the probability of survival is the probability of failure, pf, which can be expressed as:

$$p_f = 1 - p_s = P(R < Q) \quad \text{Equation 3-28}$$

To evaluate the probability of failure, pf, a single combined probability density function, g(R,Q), should be used that represents the margin of safety. This limit state function has its own unique statistics. Use of a combined probability density function, g(R,Q) is examined below to show how the probability of failure can be estimated. If R and Q are normally distributed, the limit state function g(R,Q) can be expressed as:

$$g(R,Q) = R - Q \quad \text{Equation 3-29}$$

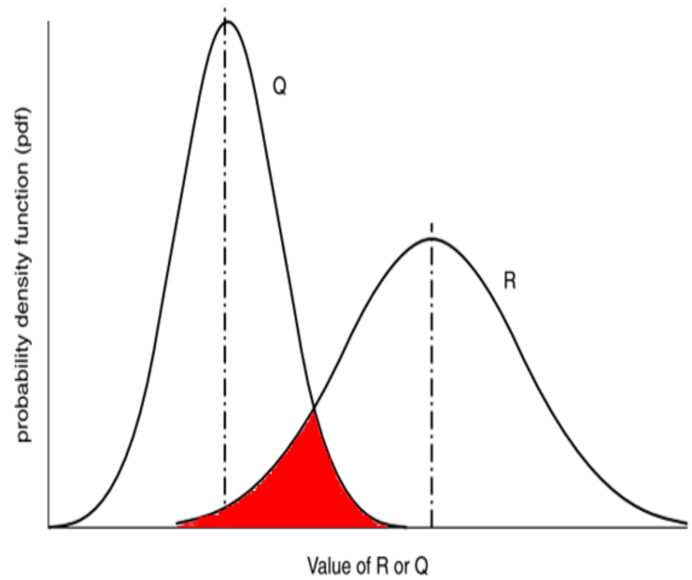



Figure 3-2 : The overlapped area is probability of failure of random variables for Q and R (Subramaniam).

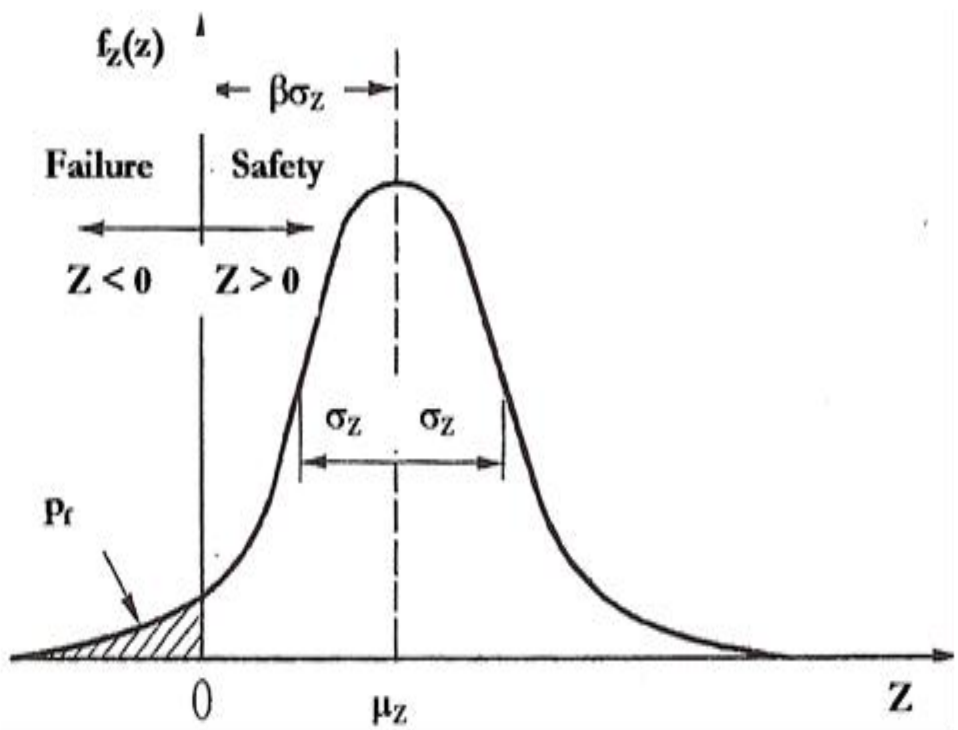



Figure 3-3:Distribution of safety margin, Z = R-Q (Melchers 2002).

If the failure function Z tracks normal distribution, the reliability of the system can be measured by reliability index, β. It was first pronounced by Cornell.

$$\beta = \mu z / (\sigma z)^{1/2} = (\mu r - \mu q) / (\sigma r - \sigma q)^{1/2} \quad \text{……………………………………. Equation 3-24}$$

μz ,μr, μq and σz, σr, σq are the mean and the standard deviation of the random variables.

For lognormally distributed R and Q, the limit state function g(R,Q) can be written as:

$$g(R,Q) = \ln R - \ln Q = \ln(R/Q) \quad \text{……………………………………. Equation 3-25}$$

In both cases, the limit state is reached when R = Q and failure occurs when g(R,Q) < 0. To determine the probability of failure, pf, it is not necessary to construct the function g(R,Q). All that is required are the mean values, R and Q , and the coefficients of variation, COVR and COVQ of the resistance, R, and load, Q, determined separately.

A commonly accepted relationship between the reliability index, β, and the probability of failure, pf, has been developed by Rosenblueth and Esteva (1972) for lognormally distributed values of R and Q using the relationship:

$$p_f = 460e^{-4.3\beta} \qquad 2 < \beta < 6 \quad \text{……………………………………. Equation 3-26}$$

$$\beta = (\ln(460/p_f))/4.3 \qquad 10^{-1} < p_f < 10^{-9} \quad \text{…………………………….Equation 3-27}$$

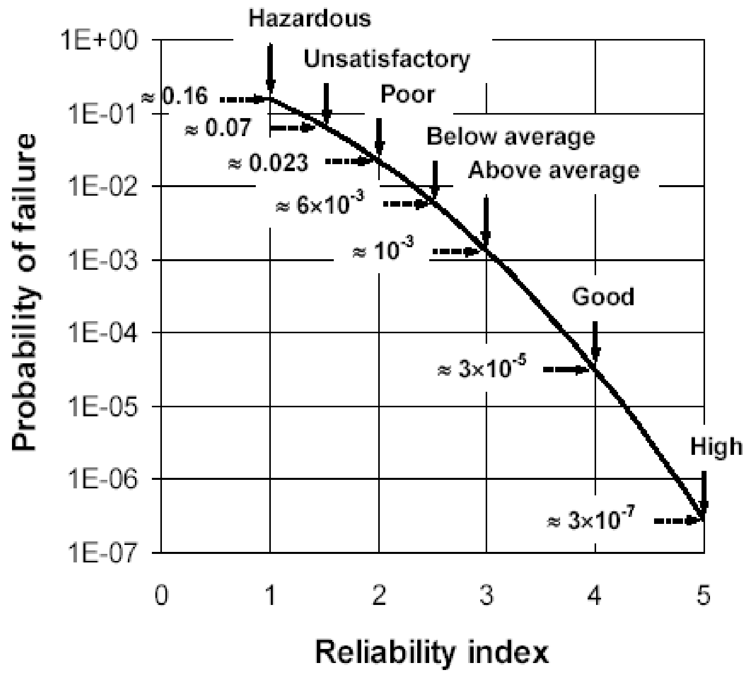



Figure 3-4: Relationship between reliability index (β) and probability of failure (pf) (US Army Corps of Engineers -1997).

The normal Gaussian distribution is the probability distribution most frequently used because of its symmetry and mathematical simplicity. It is commonly assumed to characterize many random variables. For random variable X, mean $\mu_x$ and standard deviation $\sigma_x$ probability distribution function can be expressed as;

$$f_x = N(\mu_x, \sigma_x{}^2) = \frac{\mathbf{1}}{(\mathbf{2\pi})^{\frac{1}{2}}\, \boldsymbol{x}\, \boldsymbol{\sigma}\mathbf{qu}} \cdot \boldsymbol{e}^{-\frac{1}{2}\left(\frac{\mu x - \mu qu}{\sigma qu}\right)^2} \text{ ......................................................... Equation 3-28}$$

This normal density function can be converted to standard distribution function by transforming normal variable X in to to standard normal variable Z.

$$Z = (X - \mu_x) / \boldsymbol{\sigma z} \text{ ...................................................Equation 3-29}$$

Where Z has mean 0 and standard deviation 1, i.e. N(1,0). Its corresponding probability density function is given by:

$$\Phi_Z(Z) = \frac{\mathbf{1}}{(\mathbf{2\pi})^{\frac{1}{2}}} . \boldsymbol{e}^{-\frac{1}{2}\left(\frac{Z}{1}\right)^2} \text{ .......................................................Equation 3-30}$$

Where; ΦZ(Z) is cumulative standard distribution.

## 3.4 Limit States Design Principles

### 3.4.1 Overview of limit states

The principle of safety in engineering design is that the resistance of the materials must exceed the effect of the loads, so that:

$$\text{Resistance} \geq \text{Effect of Loads} \quad \text{……………………………… Equation 3-31}$$

When applying this simple principle to design, it is essential that both sides of the inequality be evaluated for the same conditions. For example, if the effect of applied loads produces compressive stress on a soil, it is obvious that the load should be compared to the bearing resistance of the soil. When a particular loading condition reaches its limit, failure will result. Such a condition is referred to as a limit state. A **limit state** is defined as a condition beyond which a structural component, such as a foundation or other component, ceases to fulfill the function for which it is designed. Strength Limit States involve the total or partial collapse of the structure. Examples of Strength Limit States in geotechnical engineering include:

- bearing capacity failure,
- sliding, and
- overall instability.

Service Limit States affect the function of the structure under regular service conditions. Service Limit States may be reached in foundations through:

- excessive settlement,
- excessive lateral deflection,
- structural deterioration of the foundation or excessive vibration.

In this definition of a limit state, both the resistance and load are included. For example, if adequacy of bearing strength of a soil under a footing is being investigated, more than one load combination must be evaluated, especially if the footing is subjected to eccentric or inclined loads. When the bearing pressure due to the loads exceeds the bearing strength, a limit state (i.e., a Strength Limit State) is reached and failure results. Similarly, if the footing movements due to the loads exceeds the tolerable settlement, the Service Limit State is reached. An important goal, but not the only goal of the designer is to prevent a limit state from being reached.

Other goals that must be considered and balanced in the overall design are:

- function,
- appearance, and
- economy.

### 3.4.2 Design Procedures

Over the years, design procedures have been developed by engineers to provide satisfactory margins of safety. These procedures were based on the engineer's confidence in predicting the magnitude of the load and the effect of the load on the strength of the materials being provided.

#### *3.4.2.1 Determinsric design method- Allowable Stress Design (ASD)*

The design of foundations has conventionally been based on ASD. ASD is different for the Service Limit State and the Strength Limit State. For the Strength Limit State, safety is achieved in the foundation element by restricting the estimated loads (or stresses) to values less than the ultimate resistance divided by a factor of safety, FS using the relationship:

$R_n/FS \geq \sum Q_i$ ……………………………………………………………………………………Equation 3-38

where:

$R_n$ = Nominal resistance

$\sum Q_i = Q_n$ = Nominal load effect

FS = Conventional total safety factor of uncertainities

Load effects consist of dead, live and environmental load components. Environmental loads include wind, water and earthquake forces. In ASD all of these loads are assumed to have the same variability. As a result, load factors are not applied on the load combinations considered for either the strength or service limit states.

The factor of safety is a number greater than unity. The FS provides reserve strength in the event that an unusually high load occurs or in the event that the resistance is less than expected.

For the Service Limit State, unfactored loads are used to calculate deformations, and these deformations are compared to the maximum tolerable values. The advantage of ASD is its simplicity; however, it faces different shortcomings.

**Shortcomings of ASD**

In ASD, no consideration is given to the fact that various types of loads have different levels of uncertainty. In this design method dead, live, and environmental loads are all treated equally. In ASD, fixed values of design loads are selected, usually from a specification or design code. The factor of safety is applied to the resistance side of the design inequality, and the load side of the inequality is not factored.

Factors of safety in geotechnical engineering vary considerably depending on the type of problem.

- Slope Stability: $1.3 \leq FS \leq 1.5$
- Foundation Bearing Capacity: $2 \leq FS \leq 3$
- Foundation Sliding: $FS \geq 1.5$
- Foundation Overturning: $FS \geq 2.0$

Because the factor of safety chosen is based on experience and judgment, quantitative measures of risk cannot be determined for ASD.

- Does not adequately account for variability of loads and resistance. The FS is applied only to resistance. Loads are considered to be without variation (i.e. deterministic).
- Selection of a FS is subjective, and does not provide a measure of reliability in terms of probability of failure.

According to these shortcomings of ASD adoption of the reliable based design approach like load and resistance factor design (LRFD) method is desirable. Because this method overcomes mentioned deficiencies through:

- Considering variability not only in the resistance, but also in the effect of loads.
- Using the strength of the material as a basis of resistance.
- Providing a measure of safety related to probability of failure.

#### *3.4.2.2 Reliability based design approach- Load and Resistance Factor Design (LRFD)*

The American Concrete Institute (ACI) introduced a limit state design code to the ACI building code in 1956. Initially, the code did not include any resistance factors, only load factors, so the code was known as load factor design (LFD). Later resistance factors are introduced and LFD converted to LRFD.

In LRFD, the resistance side of Equation 3-40 is multiplied by a statistically-based resistance factor, φ, whose value is usually less than one. As applied to the geotechnical design of substructures, φ accounts for factors such as weaker foundation soils than expected, poor construction of the foundations, and foundation materials such as concrete, steel or wood that may not completely satisfy the requirements in the specifications.

The load components on the right side of Equation 3-41 are multiplied by their respective statistically based load factors, $\gamma_i$, whose values are usually greater than one. Because the load effect at a particular limit state involves a combination of different load types, $Q_i$, each of which has different degrees of predictability, the load factors differ in magnitude for the various load types. Therefore, the load effects can be represented by a summation of $\gamma_i Q_i$ products. If the nominal resistance is given by $R_n$, then the safety criterion can be written as:

$$R_r = \varphi R_n \geq \sum \eta_i \gamma_i Q \quad \text{.......... Equation 3-39}$$

$\eta_i$ = Load modifier to account for effects of ductility, redundancy and operational importance

$\gamma_i$ = Statistically-based load factor

$Q_i$ = Load effect

$\varphi$ = Statistically-based resistance factor

$R_n$ = Nominal resistance

For a satisfactory design, the factored nominal resistance should equal or exceed the sum of the factored load effects for a particular limit state. The value of φ chosen for a particular limit state can take into account the:

- Variability of the soil and rock properties,
- Reliability of the equations used for predicting resistance,
- Quality of the construction workmanship and quality control programs,
- Extent of soil exploration, and
- Consequence(s) of a failure.

The load factor, $\gamma_i$, chosen for a particular load type must consider the uncertainties in the:

- Magnitude and direction of loads ,
- Location of application of loads, and
- Possible combinations of loads.

**Advantages and Limitations of LRFD**

**Advantages of LRFD:**

- Accounts for variability in both resistance and load.
- Achieves relatively uniform levels of safety based on the strength of soil and rock for different limit states and foundation types.
- Provide more consistent levels of safety in the superstructure and substructure as both are designed using the same loads for predicted or target probabilities of failure.

**Limitations of LRFD:**

- The most rigorous method for developing and adjusting resistance factors to meet individual situations requires availability of statistical data and probabilistic design algorithms.
- Resistance factors vary with design methods and are not constant.

**LRFD Calibration**

The process of assigning values to resistance factors and load factors is called calibration.

A design code may be calibrated by use of :

(1) judgment,

(2) fitting to other codes,

(3) reliability theory, or

(4) a combination of approaches.

Calibration by judgment requires experience. For example, poor past performance of foundations may force a code authority to adjust the code until satisfactory results are achieved. Code parameters for structures that perform satisfactorily were accepted as correct, although this may be excessively conservative. A fundamental disadvantage of this method of calibration is that it results in non-uniform levels of conservatism.

Calibration by fitting to other codes, or simply fitting, involves using parameters (i.e., resistance factors) that would result in the same minimum permissible physical dimensions of a foundation as by ASD. Calibration by fitting does not achieve more uniform margins of safety than the ASD procedures it replaces. It does, however, make it possible to use the same loads for superstructure and foundation, and it ensures that the new code will not lead to radically different designs from the old code. Calibration by fitting with ASD can be used where there is insufficient statistical data to perform a more formal process of calibration by reliability theory.

A code can be calibrated by fitting to ASD as follows:

Divide the LRFD equation with $\eta_i = 1.0$ by the ASD equation :

$$\varphi \geq \sum \eta i \, \gamma i \, Qi / FS \times \sum Q_i$$

When $\eta_i = 1.0$, $\varphi \geq \sum \gamma i \, Qi \, / \, FS \times \sum Q_i$ …………………………………………………… Equation 3-32

If the loads consist only of dead load $Q_D$ and live load $Q_L$, then above equation becomes:

$$\varphi = (\gamma_D Q_D + \gamma_L Q_L) / FS(Q_D + Q_L)$$

$$\varphi R_n \geq \sum \gamma_i Q_i$$

For a lognormal resistance distribution, the mean value of resistance, R, can be solved as:

$$R = Q\exp\{\beta \, SQRT\, (\ln[(1+COV_R^2)(1+COV_Q^2)^{]}\} / (1+COV_Q^2)(1+COV_R^2))$$

Substituting R from EqUATION 3-42 gives the following expression for the resistance factor:

$$\varphi = \frac{\lambda_R (\Sigma \gamma_i Q_i) \sqrt{(1+COV^2_Q)/(1+COV^2_R)}}{\overline{Q} \exp\{\beta_T \sqrt{\ln[(1+COV^2_R)(1+COV^2_Q)]}\}}$$ ……………………………………… Equation 3-33

Dividing both numerator and denominator by $Q_L$, $\varphi$ can be calculated as :

$$\varphi = \frac{\lambda_R (\gamma_D Q_{D/} Q_L + \gamma_L) \sqrt{(1+COV^2 Q_D + COV^2 Q_L)/(1+COV^2_R)}}{(\lambda_{QD} Q_{D/} Q_L + \lambda_{QD}) \exp\{\beta_T \sqrt{\ln[(1+COV^2_R)(1+COV^2 Q_D + COV^2 Q_L)]}\}}$$ ………………… Equation 3-34

Where;

$\varphi$ = resistance factor

$\lambda_R$ = bias facor of resistance

$\gamma_D$ = dead load factor

$Q_D$ = dead load

$Q_L$ = live load

$\beta_T$ = target reliability index

$COV_Q$ = coeeficient of variation of load

$COV_R$ = coefficient of variation of resistance

The basic procedure adopted for calibration of the AASHTO LRFD Specifications by reliability theory employed the following steps:

Step 1: Estimate the level of reliability (which is related to the probability of success or failure) implied in the current ASD methods for analyzing foundations.

Step 2: Observe the variations of reliability levels with different span lengths, load ratios (e.g. dead to live load and other load combinations), geometry of the foundations and methods of predicting resistance.

Step 3: Select a target reliability index based on the margin of safety.

Step 4: Caculate resistance factors consistent with selected reliability index.

Based on above steps reliability index can be calculated and resistance factors will be proposed.

# CHAPTER FOUR

## 4 ANALYSIS AND DESIGN OF FOUNDATIONS

### Part I: Analysis and Design of Isolated Spread Footing

Laboratory test results

**Date**: 22/03/2018 – 13/04/2018

**A) Direct shear test**

**Location:** Arat kilo, Addis Ababa

**Project:** Residential houses

**Purpose:** This test is performed to determine the consolidated-drained shear strength of a sandy to silty soils.

**Standard Reference:** ASTM D 3080 - Standard Test Method for Direct Shear Test of Soils Under Consolidated Drained Conditions

**Significance:** To determine the shear strength parameter of a cohesionless soil.

**Equipments used:** Direct shear device, shear box, porous insers, device for shearing the specimen, shearforce measurement device, shear box bowl, controlled high humity room, trimmer or cutting ring, balances, deformation indicators, equipment for remolding or compacting specimens, miscellaneous equipment.

**Test Procedure:**

- Initial mass of soil in the pan measured.
- Diameter and height of the shear box measured and 15% of the diameter in millimeters computed.
- The shear box assembled and placed it in the direct shear device.
- The sand placed into the shear box and leveled off the top. A filter paper, a porous stone, and a top plate (with ball) were placed on top of the sand.

- The large alignment screws from the shear box removed. The gap between the shear box halves to approximately 0.025 in. opened using the gap screws, and then back out the gap screws.
- The pan of soil again weighted and the mass of soil used computed.
- The assembly of the direct shear device completed and initialized the three gauges (Horizontal displacement gage, vertical displacement gage and shear load gage) to zero.
- The vertical load (or pressure) set to a predetermined value, and then closed bleeder valve and applied the load to the soil specimen by raising the toggle switch.
- The motor with selected speed started so that the rate of shearing is at a selected constant rate, and the horizontal displacement gauge , vertical displacement gage and shear load gage readings were taken. The readings then recorded on the data sheet.
- Readings were taken continously until the horizontal shear load peaks and then falled, or the horizontal displacement reached 15% of the diameter.

**B) Specific gravity test**

**Purpose:** This test is performed to determine the specific gravity of soils.

**Standard Reference:** ASTM D 854

**Significance:** To calculate the denity of soils.

**Equipments:** pycnometer, balance, drying oven, thermometer, desiccator, entrapped air removal, hot plate or Bunsen burner, vaccum system, insulated container, funnel, pycnometer filling tube with lateral vents, sieve, blender, miscellaneous equipment .

**Test Procedure:**

- Mass of the pycnometer was verified within 0.06 g of the average calibrated mass.
- Water content of a portion of the sample determined. Then, the range of wet masses for the specific gravity specimen calculated.

- To disperse the soil about 100 mL of water put into the mixing container of a blender.The soil added and blended.
- The slurry poured into the pycnometer by using the funnel.
- The specimen dried to a constant mass in an oven at 105°C.
- The funnel placed into the pycnometer.
- The Soil Slurry prepared : Water added until the water level is between 1/3 and 1/2 of the depth of the main body of the pycnometer. The water agitated until slurry is formed. Any soil adhering to the pycnometer rinsed into the slurry.
- The Soil Slurry Deaired
- The Pycnometer filled with Water.
- The pycnometer put into a covered insulated container along with the thermometer, a beaker of deaired water.These items kept in the closed container overnight to achieve thermal equilibrium.
- Pycnometer Mass Determination : The insulated container moved near the balance. The container opened and the pycnometer removed. The pycnometer placed on an insulated block.
- The mass of pycnometer, soil, and water measured and recorded.
- The temperature of the slurry measured using the thermometer.
- Mass of Dry Soil : The mass of pan measured. The soil slurry transferred to the pan. The specimen dried to a constant mass in an oven 105°C . The dry mass of soil solids plus pan measured using the designated balance. Then, the mass of dry soil solids calculated.

**C) Laboratory test for moisture content of soil**

**Purpose:** This test is performed to determine the water content of soil.

**Standard Reference:** ASTM D 2216

**Significance:** To express the phase relationships of air, water, and solids in a given volume of material.

**Equipments:** Drying oven, balance, drying oven, specimen container, desiccator, container handling apparatus , knives, spatulas, scoops, quartering cloth, sample splitters **.**

**Test Procedure:**

- The mass of clean and dry specimen container determined and recorded.
- Representative test specimen selected.
- The moist test specimen placed in the container.
- The material dried to a constant mass at 105°C.
- The container removed from the oven after the material had dried.
- The mass of the container and oven dried material determined using same balance.
- Moisture content calculated and recorded.

**Results obtained:**

Table 4-1 : Laboratory test results for soil sample 1

| Parameter | Value |
|---|---|
| Lab-No. | 1676/10 |
| Depth (m) | 2 |
| Sample Type | Disturbed |
| Initial Specimen height (mm) | 20.00 |
| Initial Specimen area ( square cm) | 36.00 |
| Specimen Size ( mm) | 60 x 60 x20 |
| Initial volume ( cubic cm) | 72.0 |
| Specific gravity | 2.49 |
| Bulk Unit weight (gm/cc) | 1.472 |
| Moisture Content (%) | 21.91 |

Table 4-2: Shear strength parameters for sample 1

| Normal stress (kPa) | 100 | 200 | 300 | Cohesion (kPa) | Friction angle (degree) |
|---|---|---|---|---|---|
| Shear stress (kPa) | 64 | 116 | 168 | 5 | 20 |

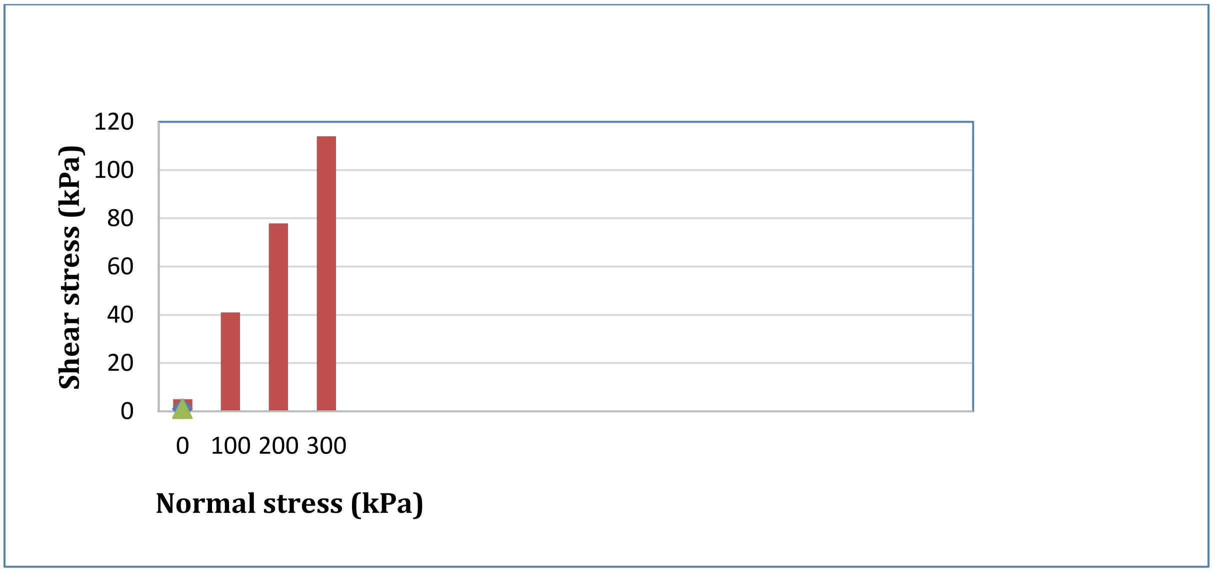



Figure 4-1 : Graphical results of normal stress versus shear stress for sample 1

Table 4-3 : Laboratory test results for soil sample 2

| Parameter | Value |
|---|---|
| Lab-No. | 1677/10 |
| Depth (m) | 2 |
| Sample type | disturbed |
| Initial specimen height (mm) | 20.00 |
| Initial specimen area ( square cm) | 36.00 |
| Specimen size ( mm) | 60 x 60 x20 |
| Initial volume ( cubic cm) | 72.0 |
| Cohesion (kPa) | 12 |
| Angle of internal friction (degree) | 28 |
| Specific gravity | 2.52 |
| Bulk Unit weight (gm/cc) | 1.498 |
| Moisture Content (%) | 23.70 |

Table 4-4: Shear strength parameters for sample 2

| Normal stress (kPa) | 100 | 200 | 300 | Cohesion (kPa) | Friction angle (degree) |
|---|---|---|---|---|---|
| Shear stress (kPa) | 64 | 116 | 168 | 12 | 28 |

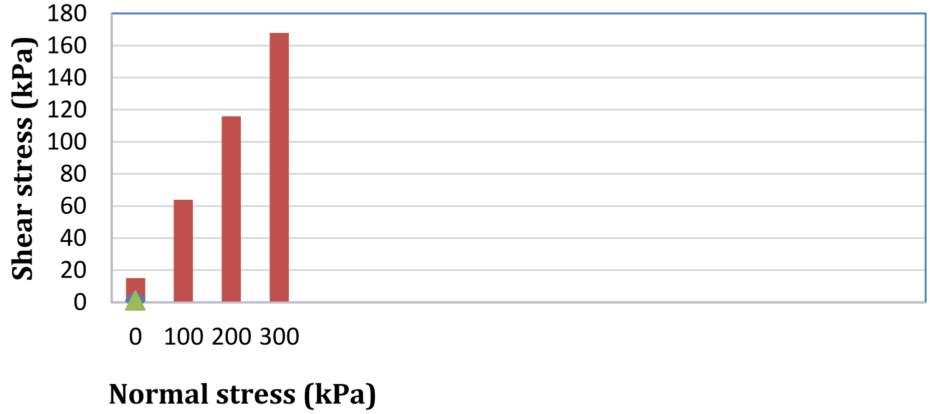



Figure 4-2 : Graphical results of normal stress versus shear stress for sample 2

Average value of the parameters considered and analysis of bearing capacity of the soil is done below.

## 4.1. Deterministic analysis methods

- Based on above test results sample 2 is taken for analysis and design of isolated spread footing and some additional necessary parameters are considered as follows;

Table 4-5 : Input parameters for design

| No. | Input Parameter | Value |
|---|---|---|
| 1 | Depth of excavation (D) | 2m |
| 2 | Average moisture content (ω) | 22.805% |
| 3 | Average dry unit weight (γd) | 12.09 kN/m$^3$ |
| 4 | Average saturated unit weight (γsat) | 14.85 kN/m$^3$ |
| 5 | Angle of friction (ϕ) | 24$^o$ |
| 6 | Cohesion (c) | 8.5 kPa |
| 7 | Poissons ratio (ν) | 0.3 |
| 8 | Length of footing (L) | 3m |
| 9 | Width of footing (B) | 2.5m |
| 10 | Total vertical loads (Qv) | 1000kN |
| 11 | Total horizontal loads (Qh) | 100kN |
| 12 | Moment along x and y direction | 70kN |

| | | |
|---|---|---|
| 13 | Factor of safety (FS) | 3 |

Average saturated unit weight ($\gamma sat$) = 1.485 x 10 x 1000 N/m$^3$

$\gamma sat$= 14850 N/m$^3$ = 14.85 kN/m$^3$

Average dry unit weight ($\gamma d$) = $\frac{\gamma sat}{1+\omega}$

$$\gamma d = \frac{14.85}{1+0.22805} = 12.09 \text{ kN/m}^3$$

Factor of safety (FS) is considered 3 based on commonly used foundation design practices. Length, width and also loads are taken randomly from soil sample location. Here construction of residential building is started and loading conditions considered from highlight observations.

### 4.1.1 Terzaghi's bearing capacity equation

$e_B = e_L =$ M /V = 70/1000= 0.07

B' = B- 2e = 2.5- 2 x 0.08 = **2.34m**

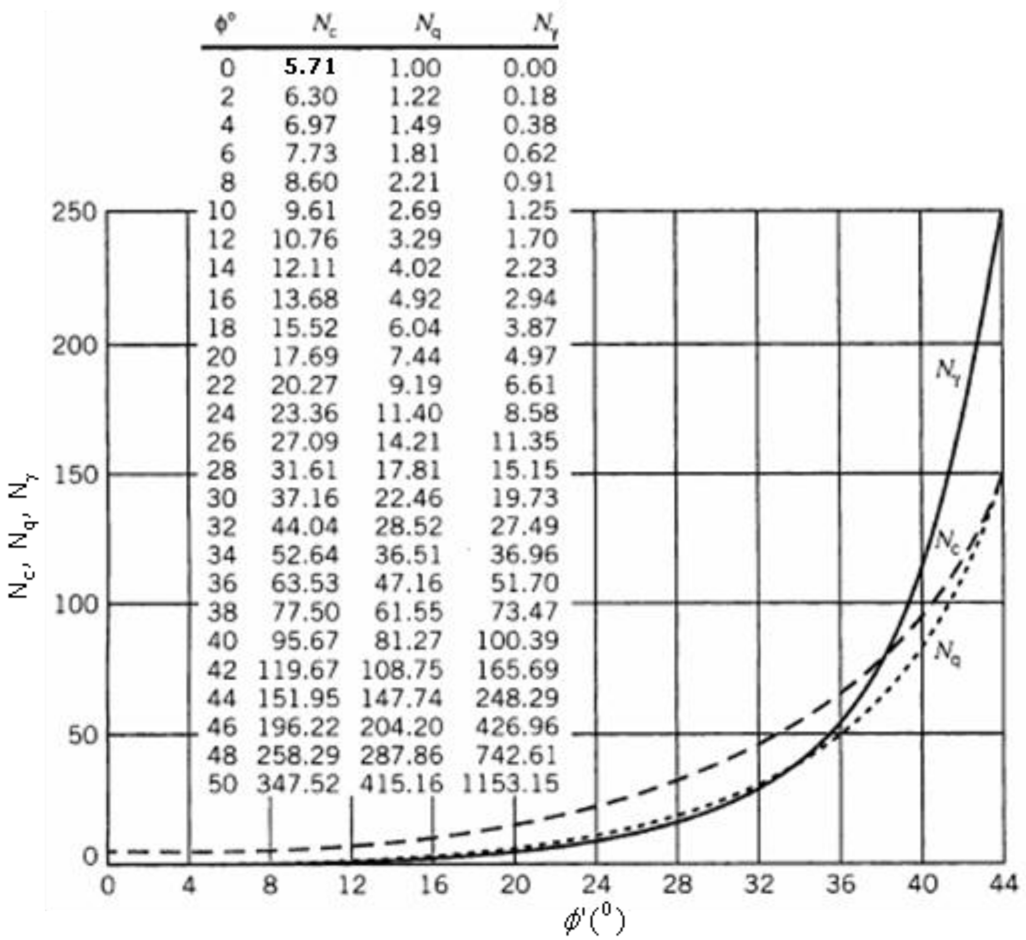
L' = L- 2e = 3- 2 x 0.08 = **2.84m**

Qult =k1 cNc + qNq + k2 $\gamma$B' N$\gamma$

For strip footing, $k_1$ = 1. &

$k_2$ = 0.5

Cohesion, c= undrained shear strength =8.5kPa




| $\phi°$ | $N_c$ | $N_q$ | $N_\gamma$ |
|---|---|---|---|
| 0 | 5.71 | 1.00 | 0.00 |
| 2 | 6.30 | 1.22 | 0.18 |
| 4 | 6.97 | 1.49 | 0.38 |
| 6 | 7.73 | 1.81 | 0.62 |
| 8 | 8.60 | 2.21 | 0.91 |
| 10 | 9.61 | 2.69 | 1.25 |
| 12 | 10.76 | 3.29 | 1.70 |
| 14 | 12.11 | 4.02 | 2.23 |
| 16 | 13.68 | 4.92 | 2.94 |
| 18 | 15.52 | 6.04 | 3.87 |
| 20 | 17.69 | 7.44 | 4.97 |
| 22 | 20.27 | 9.19 | 6.61 |
| 24 | 23.36 | 11.40 | 8.58 |
| 26 | 27.09 | 14.21 | 11.35 |
| 28 | 31.61 | 17.81 | 15.15 |
| 30 | 37.16 | 22.46 | 19.73 |
| 32 | 44.04 | 28.52 | 27.49 |
| 34 | 52.64 | 36.51 | 36.96 |
| 36 | 63.53 | 47.16 | 51.70 |
| 38 | 77.50 | 61.55 | 73.47 |
| 40 | 95.67 | 81.27 | 100.39 |
| 42 | 119.67 | 108.75 | 165.69 |
| 44 | 151.95 | 147.74 | 248.29 |
| 46 | 196.22 | 204.20 | 426.96 |
| 48 | 258.29 | 287.86 | 742.61 |
| 50 | 347.52 | 415.16 | 1153.15 |

Figure 4-3: Terzhagi's bearing capacity factors

Bearing capacity factors based on Terzaghi formula for $\phi = 24^o$;

$N_c = 23.36$ ,

$N_q = 11.40$ &

$N\gamma = 8.58$

$q= \gamma D_f= 12.09 \times 2 =24.18$kPa

Therefore ultimate bearing capacity, Qult = 8.5 x 23.36+24.18 x 11.4+0.5 x 12.09 x 2.34 x 8.58

Qult = 198.56+275.65+121.37

**Qult = 595.58kPa**

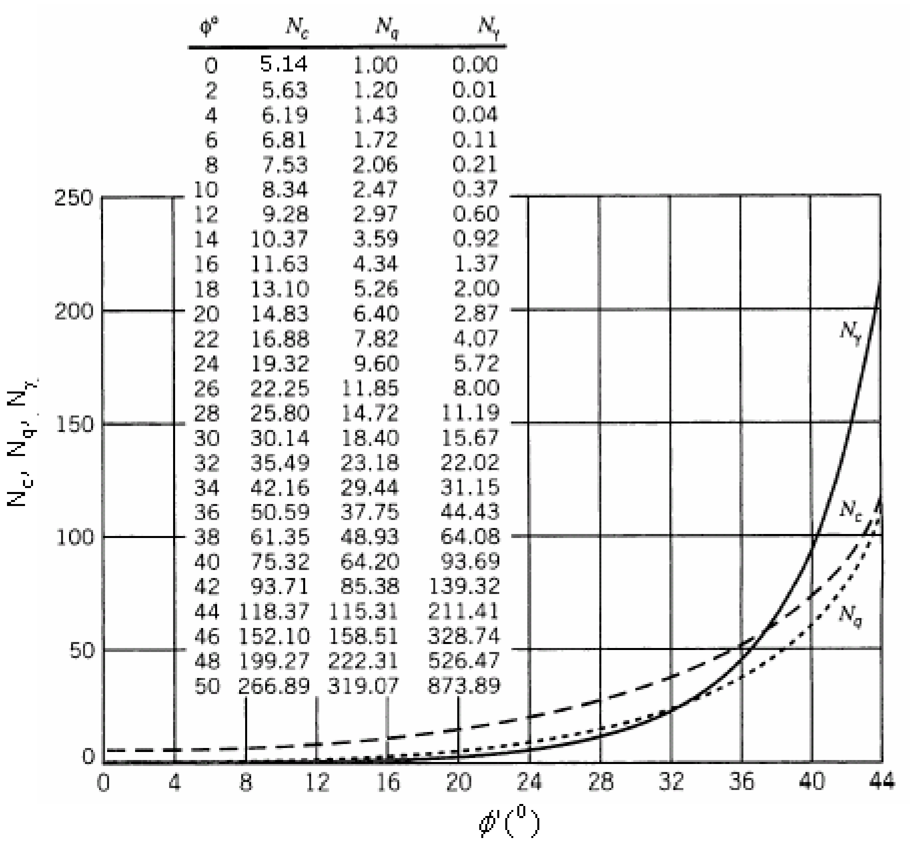
Allowable bearing capacity, Qall = Qult/FS

Qall = 595.58/3

**Qall = 198.53kPa**

### 4.1.2 Meyerhof's bearing capacity theory

Qult= c Nc Sc dc ic + q Nq Sq dq iq + ½ γ B Nγ Sγ dγ iγ




| $\phi^\circ$ | $N_c$ | $N_q$ | $N_\gamma$ |
|---|---|---|---|
| 0 | 5.14 | 1.00 | 0.00 |
| 2 | 5.63 | 1.20 | 0.01 |
| 4 | 6.19 | 1.43 | 0.04 |
| 6 | 6.81 | 1.72 | 0.11 |
| 8 | 7.53 | 2.06 | 0.21 |
| 10 | 8.34 | 2.47 | 0.37 |
| 12 | 9.28 | 2.97 | 0.60 |
| 14 | 10.37 | 3.59 | 0.92 |
| 16 | 11.63 | 4.34 | 1.37 |
| 18 | 13.10 | 5.26 | 2.00 |
| 20 | 14.83 | 6.40 | 2.87 |
| 22 | 16.88 | 7.82 | 4.07 |
| 24 | 19.32 | 9.60 | 5.72 |
| 26 | 22.25 | 11.85 | 8.00 |
| 28 | 25.80 | 14.72 | 11.19 |
| 30 | 30.14 | 18.40 | 15.67 |
| 32 | 35.49 | 23.18 | 22.02 |
| 34 | 42.16 | 29.44 | 31.15 |
| 36 | 50.59 | 37.75 | 44.43 |
| 38 | 61.35 | 48.93 | 64.08 |
| 40 | 75.32 | 64.20 | 93.69 |
| 42 | 93.71 | 85.38 | 139.32 |
| 44 | 118.37 | 115.31 | 211.41 |
| 46 | 152.10 | 158.51 | 328.74 |
| 48 | 199.27 | 222.31 | 526.47 |
| 50 | 266.89 | 319.07 | 873.89 |

Figure 4-4: Meyerhof's bearing capacity factors

Based on Meyerhof's equation for ϕ = 24°;

- Bearing capacity factors:

  Nc = (Nq -1)cot ϕ = 19.32

  Nq $= e^{\Pi \tan\phi}\, tan^2(45 + \phi/2)$ = 9.60

  Nγ = (Nq -1)tan(1. 4ϕ) = 5.72

- Shape factors :

  Nϕ=(1+sin ϕ)/(1-sin ϕ)=(1+sin24°)/(1-sin24°)=2.37

$Sc = 1+0.2\ x\ N\phi \left(\frac{B}{L}\right) = 1+0.2 \text{ x } 2.37 \left(\frac{2.5}{3}\right) = 1.395$

$Sq = 1+0.1\ x\ N\phi \left(\frac{B}{L}\right) = 1+0.1 \text{ x } 2.37 \left(\frac{2.5}{3}\right) = 1.198$

$S\gamma = Sq = 1.198$ , for $\phi \geq 10^{o}$

- Depth factors :

$N\varphi=(1+\sin\phi)/(1-\sin\phi)=(1+\sin 24^{o})/(1-\sin 24^{o})=2.37$

$dc = 1+0.2\left(\frac{Df}{B}\right)(1+\tan\phi) = 1+0.2\left(\frac{2.0}{2.5}\right)(1+\tan 24) = 1.23$

$dq = 1+0.1\left(\frac{Df}{B}\right)(1+\tan\phi) = 1+0.1\left(\frac{2.0}{2.5}\right)(1+\tan 24) = 1.12$

$d\gamma = dq = 1.12$ , for $\phi \geq 10^{o}$

- Inclination factors :

$\alpha = Qv/Qh = 1000/100 = 10^{o}$

$ic = (1 - {}^{\alpha}/90)^2 = (1-10/90)^2 = 0.79$

$iq = ic$ for any $\phi$

$iq = 0.79$

$i\gamma = (1 - {}^{\alpha}/\phi)^2 = (1-10/24)^2 = 0.34$

Now insert all above values to Meyerhof bearing capacity equation gives;

$Qult= c\ Nc\ Sc\ dc\ ic + q\ Nq\ Sq\ dq\ iq + ½\ \gamma\ B\ N\gamma\ S\gamma\ d\gamma\ i\gamma$

Qult=8.5 x 19.32 x 1.395 x 1.23 x 0.79 + 24.18 x 9.60 x 1.198 x 1.12 x 0.79 + ½ x 12.09 x 2.5 x 5.72 x 1.198 x 1.12 x 0.34

Qult = 222.60+246.05+39.44

**Qult = 508.09kPa**

Allowable bearing capacity, Qall = Qult/FS

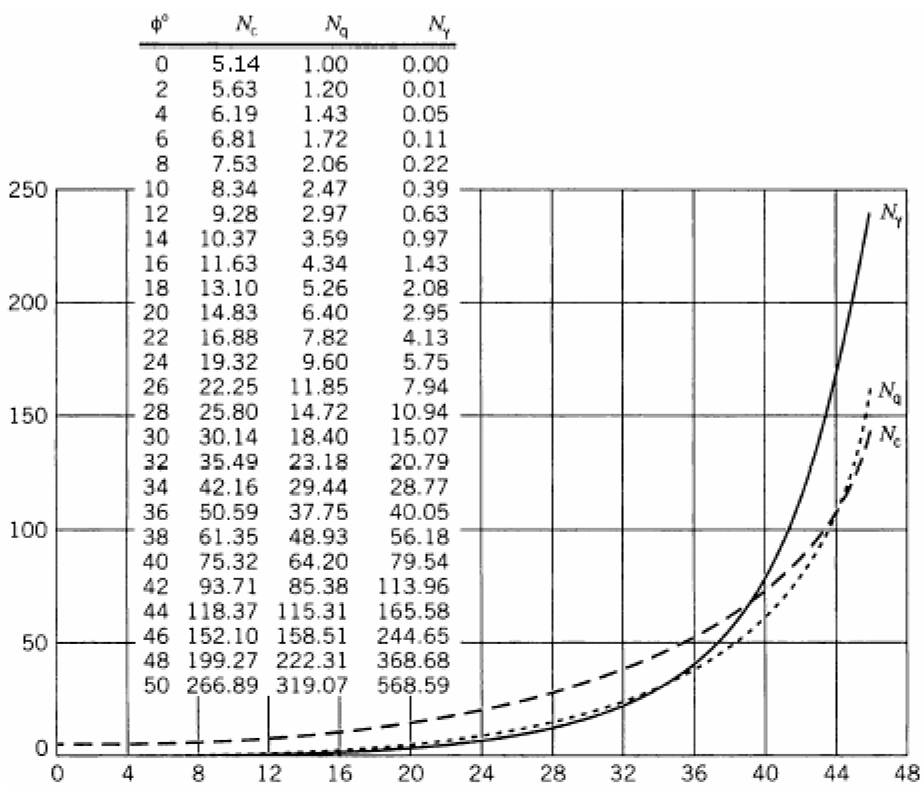
Qall = 508.09 /3

**Qall = 169.36kPa**

### 4.1.3 Hansen's bearing capacity theory

According to Hansen, the ultimate bearing capacity equation is given as;

$$Q_{ult} = c\,N_c\,S_c\,d_c\,i_c + q\,N_q\,S_q\,d_q\,i_q + \tfrac{1}{2}\,\gamma\,B\,N_\gamma\,S_\gamma\,d_\gamma\,i_\gamma$$



| $\phi°$ | $N_c$ | $N_q$ | $N_\gamma$ |
|---|---|---|---|
| 0 | 5.14 | 1.00 | 0.00 |
| 2 | 5.63 | 1.20 | 0.01 |
| 4 | 6.19 | 1.43 | 0.05 |
| 6 | 6.81 | 1.72 | 0.11 |
| 8 | 7.53 | 2.06 | 0.22 |
| 10 | 8.34 | 2.47 | 0.39 |
| 12 | 9.28 | 2.97 | 0.63 |
| 14 | 10.37 | 3.59 | 0.97 |
| 16 | 11.63 | 4.34 | 1.43 |
| 18 | 13.10 | 5.26 | 2.08 |
| 20 | 14.83 | 6.40 | 2.95 |
| 22 | 16.88 | 7.82 | 4.13 |
| 24 | 19.32 | 9.60 | 5.75 |
| 26 | 22.25 | 11.85 | 7.94 |
| 28 | 25.80 | 14.72 | 10.94 |
| 30 | 30.14 | 18.40 | 15.07 |
| 32 | 35.49 | 23.18 | 20.79 |
| 34 | 42.16 | 29.44 | 28.77 |
| 36 | 50.59 | 37.75 | 40.05 |
| 38 | 61.35 | 48.93 | 56.18 |
| 40 | 75.32 | 64.20 | 79.54 |
| 42 | 93.71 | 85.38 | 113.96 |
| 44 | 118.37 | 115.31 | 165.58 |
| 46 | 152.10 | 158.51 | 244.65 |
| 48 | 199.27 | 222.31 | 368.68 |
| 50 | 266.89 | 319.07 | 568.59 |



Figure 4-5: Hansen’s bearing capacity factors

Based on Hansen's equation for ϕ = 24°;

- ➢ Bearing capacity factors:

$N_c = (N_q - 1)\cot\phi = 19.32$

$N_q = e^{\Pi \tan\phi}\ tan^2(45 + \phi/2) = 9.60$

$N_\gamma = 1.5\,(N_q - 1)\tan\varphi$

$N_\gamma = 1.5(14.72 - 1)\tan 24^o$

$N_\gamma = 5.75$

- Shape factors :

$$Sc = 1 + \left(\frac{Nq}{Nc}\right)\left(\frac{B}{L}\right) = 1+\left(\frac{9.60}{19.32}\right)\left(\frac{2.5}{3}\right) = 1.41$$

$$Sq = 1 + \left(\frac{B}{L}\right)\tan\phi = 1+\left(\frac{2.5}{3}\right)\tan 24 = 1.37$$

$$S\gamma = 1 - 0.4\left(\frac{B}{L}\right) = 1- 0.4\left(\frac{2.5}{3}\right) = 0.67$$

- Depth factors :

$$dc = 1+0.4\left(\frac{Df}{B}\right) = 1+0.4\left(\frac{2}{2.5}\right) = 1.32$$

$$dq = 1+2\tan\phi\,(1-\sin\phi)^2\left(\frac{Df}{B}\right) = 1+2\tan 24\,(1-\sin 24)^2\left(\frac{2}{2.5}\right) = 1.25$$

$d\gamma = 1$ , for any $\phi$

- Inclination factors :

$$iq = (1-0.5\left(\frac{Qh}{Qv+Af \text{ x } Ca \text{ x } \cot\varphi}\right))^5$$

Where;

Qh = horizontal component of inclined load

Qv = vertical component of inclined load

Ca = unit adhesion on base of footing = 1

Af = Effective contact area of footing = 2.34 x 2.84 = 6.65 $m^2$

Insert these parameters' value to above equation

$$iq = (1- 0.5\left(\frac{100}{1000+6.65 \text{ x } 1 \text{ x } \cot 24}\right))^5 = 0.78$$

$$ic = iq - \left(\frac{1-iq}{Nq-1}\right) = 0.78 - \left(\frac{1-0.78}{9.60-1}\right) = 0.75$$

$$i\gamma = (1- 0.7\left(\frac{Qh}{Qv+Af \text{ x } Ca \text{ x } \cot\varphi}\right))^5$$

$$i\gamma = (1- 0.7\left(\frac{100}{1000+6.65 \text{ x } 1 \text{ x } \cot 24}\right))^5 = 0.70$$

Now insert all above values to Hansen bearing capacity equation gives;

Qult=8.5 x 19.32 x 1.41 x 1.32 x 0.75 + 24.18 x 9.60 x 1.37 x 1.25 x 0.78 + ½ x 12.09 x 2.5 x 5.75 x 0.67 x 1 x 0.70

Qult=229.23 + 310.06 + 40.75

**Qult = 580.04kPa**

Allowable bearing capacity, Qall = Qult/FS

Qall = 580.04/3

**Qall = 193.35kPa**

### 4.1.4 Vesic's bearing capacity theory

According to Vesic, the ultimate bearing capacity equation is given as;

$$Q_{ult} = c\, N_c\, S_c\, d_c\, i_c + q\, N_q\, S_q\, d_q\, i_q + \tfrac{1}{2}\, \gamma\, B\, N_\gamma\, S_\gamma\, d_\gamma\, i_\gamma$$

From Vesic's equation for $\phi = 24^o$;

- Bearing capacity factors:

$$N_c = (N_q - 1)\cot\phi = 19.32$$

$$N_q = e^{\Pi \tan\phi}\ tan^2(45 + \phi/2) = 9.60$$

$$N_\gamma = 2\,(N_q + 1)\tan\phi$$

$$N_\gamma = 2\,(9.60 + 1)\tan 24^o$$

$$N_\gamma = 9.44$$

- Shape factors :

$$S_c = 1 + \left(\frac{N_q}{N_c}\right)\left(\frac{B}{L}\right) = 1 + \left(\frac{9.60}{19.32}\right)\left(\frac{2.5}{3}\right) = 1.41$$

$$S_q = 1 + \left(\frac{B}{L}\right)\tan\phi = 1 + \left(\frac{2.5}{3}\right)\tan 24 = 1.37$$

$$S_\gamma = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4\left(\frac{2.5}{3}\right) = 0.67$$

- Depth factors :

$$d_c = 1 + 0.4\left(\frac{D_f}{B}\right) = 1 + 0.4\left(\frac{2}{2.5}\right) = 1.32$$

$$d_q = 1 + 2\tan\phi\,(1 - \sin\phi)^2\left(\frac{D_f}{B}\right) = 1 + 2\,x\,\tan 24\ (1 - \sin 24)^2\left(\frac{2}{2.5}\right) = 1.25$$

$d_\gamma = 1$, for any $\phi$

- Inclination factors :

$i_c$ value is same as Hansen = 0.75

$$i_q = \left(1 - \left(\frac{Q_h}{Q_v + A_f \times C_a \times \cot\phi}\right)\right)^m$$

$m = \left(\frac{2+B/L}{1+B/L}\right) = \left(\frac{2+2.5/3}{1+2.5/3}\right) = 1.5$

$iq = (1- \left(\frac{100}{1000+6.65 \text{ x } 1 \text{ x } \cot 24}\right))^{1.5}$

$iq = 0.86$

$i\gamma = (1- \left(\frac{Qh}{Qv+Af \text{ x } Ca \text{ x } \cot \phi}\right))^{m+1}$

$i\gamma = (1- \left(\frac{100}{1000+6.65 \text{ x } 1 \text{ x } \cot 24}\right))^{2.5}$

$i\gamma = 0.77$

Now insert all above values to Vesic bearing capacity equation gives;

Qult=c Nc Sc dc ic + q Nq Sq dq iq + ½ γ B Nγ Sγ dγ iγ

Qult=8.5 x 19.32 x 1.41 x 1.32 x 0.75 + 24.18 x 9.60 x 1.37 x 1.25 x 0.86 + ½ x 12.09 x 2.5 x 9.44 x 0.67 x 1 x 0.77

Qult = 229.23 + 341.87 + 73.60

**Qult = 644.70kPa**

Allowable bearing capacity, Qall = Qult/FS

Qall = 644.70/3

**Qall = 214.90kPa**

### 4.1.5 Ethiopian standard with Euro Norms (ES EN 1997 -2015) bearing capacity calculation

Bearing resistance using ES EN is calculated as;

R/A= c Nc Sc bc ic + q Nq Sq bq iq + ½ γ B Nγ Sγ bγ iγ

Bearing capacity factors:

Nc = (Nq -1) cotϕ = 19.32

$Nq = e^{\Pi \tan\phi} \, tan^2(45 + \phi/2) = 9.60$

Nγ = 2 (Nq -1) tanϕ = 7.66

- Shape factors :

$Sq = 1+ \sin\phi \left(\frac{B}{L}\right) = 1 + \sin 24 \left(\frac{2.5}{3}\right) = 1.34$

$Sc = (Sq\ Nq-1)/(Nq-1) = (1.34 \text{ x } 9.60 - 1)/(9.60 - 1) = 1.38$

$S\gamma = 1 - 0.3\ (\frac{B}{L}) = 1 - 0.3 \text{ x } (\frac{2.5}{3}) = 0.75$

- Depth factors :

Inclination of foundation base, $\alpha = Qv/Qh = 1000kN/100kN = 10^o$

Changing to radian=$\pi \text{ x } \alpha/180 = 3.14 \text{ x } 10/180 = 0.17$

$bq = b\gamma = (1 - \alpha \text{ x } \tan\phi)^2 = (1 - 0.17 \text{ x } \tan 24)^2 = 0.85$,

$bc = bq - (1-bq)/(Nc \text{ x } \tan\phi) = 0.85 - (1 - 0.85)/(19.32 \text{ x } \tan 24) = 0.83$

- Inclination factors :

$Af = B' \text{ x } L' = 2.34 \text{ x } 2.84 = 6.65m^2$

$$m = \left(\frac{2+B'/L'}{1+B'/L'}\right) = \left(\frac{2+2.34/2.84}{1+2.34/2.84}\right) = 1.55$$

$$iq = (1 - \left(\frac{Qh}{Qv+Af \text{ x } Ca \text{ x } \cot\phi}\right))^m$$

$$iq = (1 - \left(\frac{100}{1000+6.65 \text{ x } 1 \text{ x } \cot 24}\right))^{1.55}$$

$iq = 0.85$

$ic = iq - (1-iq)/(Nc \text{ x } \tan\phi)$

$ic = 0.85-(1-0.85)/(19.32 \text{ x } \tan 24)$

$ic = 0.83$

$$i\gamma = (1 - \left(\frac{Qh}{Qv+Af \text{ x } Ca \text{ x } \cot\phi}\right))^{m+1}$$

$$i\gamma = (1 - \left(\frac{100}{1000+6.65 \text{ x } 1 \text{ x } \cot 24}\right))^{2.55}$$

$i\gamma = 0.77$

Now insert all above values to ES EN bearing capacity equation gives;

$Qult = c\ Nc\ Sc\ bc\ ic + q\ Nq\ Sq\ bq\ iq + ½\ \gamma\ B\ N\gamma\ S\gamma\ b\gamma\ i\gamma$

$Qult = 8.5 \text{ x } 19.32 \text{ x } 1.38 \text{ x } 0.83 \text{ x } 0.83 + 24.18 \text{ x } 9.60 \text{ x } 1.34 \text{ x } 0.85 \text{ x } 0.85 + ½ \text{ x } 12.09 \text{ x } 2.5 \text{ x } 7.66 \text{ x } 0.75 \text{ x } 0.85 \text{ x } 0.77$

$Qult = 156.12 + 224.73 + 56.82$

**Qult = 437.67kPa**

Allowable bearing capacity, $Q_{all} = Q_{ult}/FS$

$$Q_{all} = 437.67/3$$

$$\mathbf{Q_{all} = 145.89kPa}$$

Table 4-6 : Summary of bearing capacity result based on deterministic methods

| **Method of Equation** | **Ultimate Bearing Capacity Value (kPa)** | **Allowable bearing capacity value (kPa)** |
|---|---|---|
| Terzaghi equation | 595.58 | 198.53 |
| Meyerhof | 508.09 | 169.36 |
| Hansen | 580.04 | 193.35 |
| Vesic | 644.70 | 214.90 |
| ES EN 1997 -2015 | 437.67 | 145.89 |

## 4.2 Reliability based analysis methods of the bearing capacity problem

Soil sample properties: Bearing capacity analysis for isolated spread footing design with effective friction angle and cohesion as the input random variables.

Table 4-7 : Input parameters for reliability based design methods

| No. | Input Parameter | Value |
|---|---|---|
| 1 | Depth (D) | 2m |
| 3 | Average dry unit weight (γd) | 12.09 kN/m$^3$ |
| 4 | Average saturated unit weight (γsat) | 14.85 kN/m$^3$ |
| 5 | Average angle of friction (ϕ) | 24$^o$ |
| 6 | Average cohesion (c) | 8.5kPa |
| 7 | Poissons ratio (ν) | 0.3 |
| 8 | Length of footing (L) | 3m |
| 9 | Width of footing (B) | 2.5m |
| 10 | Coefficient of variation for ϕ | 14% |
| 11 | Coefficient of variation for c | 50% |
| 12 | Standard deviation of ϕ (σϕ) | 3.36$^o$ |
| 13 | Standard deviation of c (σc) | 4.25 |

Coefficient of variations for friction angle and cohesion are considered based on literatures review. Standard deviations of these soil parameters are calculated using mean value and coefficient of variations i.e.

- $\mu c' = 8.5$ kN/m$^2$, COVc=50%

  $\sigma c' = \mu c' * COVc = 4.25$

- $\mu\phi' = 24°$, COVϕ=14%

  $\sigma\phi' = \mu\phi' * COV\phi = 3.36°$

### 4.2.1 First Order Second Moment Method (FOSM)

*4.2.1.1 Case I: FOSM results for friction angle as random variable*

Considering Terzaghi´s bearing capacity formula as a function of tanϕ´, the Taylor's series expansion for the bearing capacity about the mean value μtanϕ´ will be:

$qu \approx qu(\mu\tan\phi') + (\tan\phi' - \mu\tan\phi').\partial qu/\partial\tan\phi'$ ........................................ Equation 4-1

$\mu qu(\tan\phi') = qu(\tan\phi' = \mu\tan\phi')$ ................................................................ Equation 4-2

$Var(qu) = Var(\tan\phi').(\partial qu/\partial\tan\phi')^2$ ........................................................ Equation 4-3

Bearing capacity factors Nq, Nc, Nγ values for $\phi = 24^o$;

$N_c = 23.36$ ,

$N_q = 11.40$ &

$N\gamma = 8.58$

$qu = c\,Nc + qNq + 0.5\,\gamma\,B\,N\gamma$

Substituting bearing capacity factors equations to Terzaghi equation as a function of tan$\varphi$ gives:

$$qu = \frac{c}{\tan\phi}\left\{e^{\pi tan\phi}\left[tan\phi + \left(1 + tan^2\phi\right)^{\frac{1}{2}}\right]^2 - 1\right\} + q.\,e^{\pi tan\phi}\left[tan\phi' + \left(1 + tan^2\phi'\right)^{\frac{1}{2}}\right]^2 +$$
$$1.5.\gamma tan\phi\left\{e^{\pi tan\phi}\left[tan\phi + \left(1 + tan^2\phi\right)^{\frac{1}{2}}\right]^2 - 1\right\} \quad \text{........................................ Equation 4-4}$$

qu = 164.25 + 232.21 + 69.47

**qu = 465.93kPa**

Table 4-8 : Soil sample input parameters and bearing capacity calculation

| Input data | Determinstic value |
|---|---|
| Width of footing (B) | 2.5m |
| Depth (D) | 2m |
| Avrage dry unit weight ($\gamma d$) | 12.09 $kN/m^3$ |
| Average cohesion (c) | $8.5kN/m^2$ |
| Average angle of friction ($\phi$) | $24^o$ |
| tan $\phi$ | 0.445 |
| Nc | 23.36 |
| Nq | 11.40 |
| N$\gamma$ | 8.58 |
| Surcharge (qo) | $24.18kN/m^2$ |
| Ultimate bearing capacity (Qult) | 465.93 |

The first derivative of the bearing capacity computed analytically with respect to tanϕ´ is:

$$\frac{\partial qf}{\partial tan\varphi'} =$$

$$=-22.5+ c'\left\{\frac{1}{tan^2\phi'} - \frac{1}{tan^2\phi'} e^{\pi tan\phi'}\left[tan\phi' + \left(1 + tan^2\phi'\right)^{\frac{1}{2}}\right]^2 + \frac{\pi}{tan^2\phi'} . e^{\pi tan\phi'}\left[tan\phi' + \left(1 + tan^2\phi'\right)^{\frac{1}{2}}\right]^2 + 4e^{\pi tan\phi'}\left[tan\phi' + (1 + tan^2\phi')^{\frac{1}{2}}\right].[1 + \frac{1}{2}(1 + tan^2\phi')^{\frac{-1}{2}}]\right\} +$$

$$q.\left\{\pi e^{\pi tan\phi'}\left[tan\phi' + \left(1 + tan^2\phi'\right)^{\frac{1}{2}}\right]^2 + 4e^{\pi tan\phi'} tan\phi'\left[tan\phi' + (1 + tan^2\phi')^{\frac{1}{2}}\right].[1 +\right.$$

$$\frac{1}{2}(1+tan^2\phi')^{\frac{-1}{2}}]\Big\} + 1.5\gamma.\Big\{e^{\pi tan\phi'}[tan\phi' + \Big(1+tan^2\phi')^{\frac{1}{2}}\Big]^2 + \pi tan\phi' e^{\pi tan\phi'}[tan\phi' + \Big(1+tan^2\phi')^{\frac{1}{2}}\Big]^2 + 4tan^2\phi' e^{\pi tan\phi'}\Big[tan\phi' + (1+tan^2\phi')^{\frac{1}{2}}\Big].[1+\frac{1}{2}(1+tan^2\phi')^{\frac{-1}{2}}]\Big\}$$

……………………………………..……………….….. Equation 4-5

Substitute the numerical values of the unit weight, surcharge, and cohesion and mean value of tanϕ´ gives:

$\frac{\partial q}{\partial tan\phi'}$ = 1211.165 + 1120.797 + 548.348 = 2880.31

Var (qu) = Var$(tan\phi) x$ $(\frac{\partial q}{\partial tan\phi'})^2$ =$(0.06\ )^2$ x $(2880.31)^2$

Var(qu) = 29866.27 $(kN/m^2)^2$

$\boldsymbol{\sigma\ qu = 172.82\ kN/m^2}$

Table 4-9: Statistical values of qu predicted by FOSM method for friction angle as random variable

| μqu(KN/$m^2$) | σ qu(KN/$m^2$) | COVqu |
|---|---|---|
| 465.93 | 172.82 | 0.37 |

Based on above results probability density function for FOSM will be as follows;

PDFx = $\frac{\mathbf{1}}{(2\pi)^{\frac{1}{2}}\ x\ \sigma qu} .\ e^{-\frac{\mathbf{1}}{\mathbf{2}}(\frac{\boldsymbol{\mu x - \mu qu}}{\boldsymbol{\sigma qu}})^{\mathbf{2}}}$ ……………………………………………………………… Equation 4-6

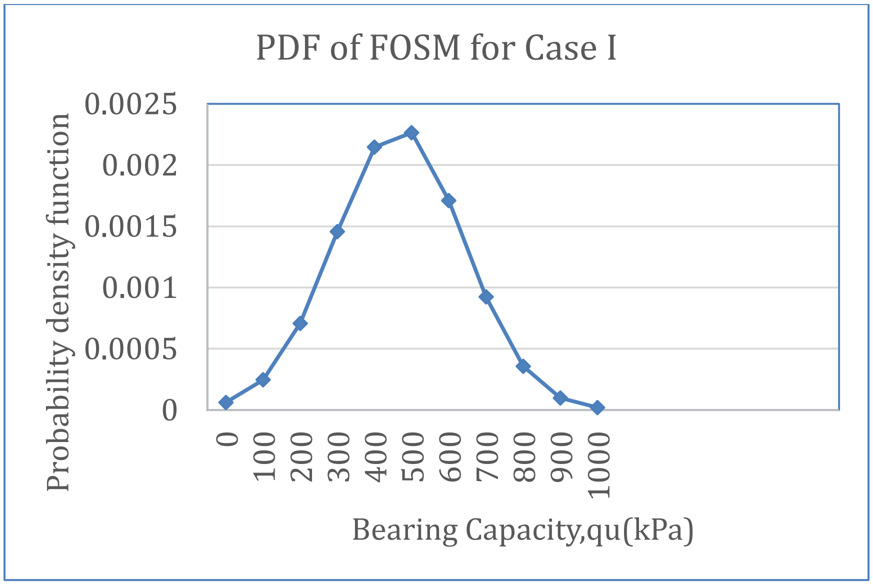



Figure 4-6 : PDF of FOSM for friction angle as random variable

*4.2.1.2 Case II: FOSM results by considering uncorrelated random variables*

Considering now the bearing capacity as a function of both soil parameters tanϕ´ and cohesion, the Taylor's series expansion for the bearing capacity about the mean values μtanϕ´ and μc´, truncated after the first order terms, is given by;

$qu \approx qu(\mu\tan\phi', \mu c') + (\tan\phi' - \mu\tan\phi').\partial qu/\partial\tan\phi' + (c' - \mu c').\partial qu/\partial c'$ ………… Equation 4-7

Where the derivatives are evaluated at the mean values μtanϕ´ and μc´. The mean value and variance of the bearing capacity are;

$\mu qu(\tan\phi', c') = qu(\mu\tan\phi' = \tan\phi', \mu c' = c')$ ……………………………………………….. Equation 4-8

$Var(qu) = Var(\tan\phi').\ (\partial qu/\partial tan\ \phi')^2 + Var(c').(\partial qu/\partial c')^2$ ……………..………… Equation 4-9

Substituting the mean value of cohesion and tanϕ´, the expected bearing capacity value is μqu = 465.93 kN/$m^2$.

The first derivatives of the bearing capacity computed analytically with respect to cohesion

$\frac{\partial qf}{\partial c'} = \frac{1}{tan\phi'}\left\{e^{\pi tan\phi'}[tan\phi' + (1 + tan^2\phi')^{\frac{1}{2}}]^2 - 1\right\}$ …………………...……….…….. Equation 4-10

Substituting the numerical values for the mean value of the cohesion and friction angle and take average coefficient of variation for cohesion according to litratures.

$\frac{\partial qu}{\partial \tan \phi'}$ = 2880.31

$\partial$qu/ $\partial c'$= 19.32 &

Var(c) = $(\sigma c)^2 = (\sigma c)^2 = (4.25)^2$ =18.06

Hence the variance and the standard deviation values are;

Var(qu) = $(29866.27+18.06 \times 19.32^2)(kN/m^2)^2$

Var(qu) =36607.389 $(kN/m^2)^2$

$\sigma$ qu = 191.33 $kN/m^2$

Cov qu = $\frac{\sigma\ qu}{\mu qu}$ = 0.41

Table 4.10 below shows the statistical values of the bearing capacity for uncorrelated soil parameters. The mean value is exactly the same of case I because in both cases the numerical value of the soil parameters is unchanged. The standard deviation and, consequently, the COVqu are increased.

Table 4-10 : Statistical values of qu predicted by FOSM method for uncorrelated soil parameters

| $\mu qu(kN/m^2)$ | $\sigma\ qu(kN/m^2)$ | COVqu |
|---|---|---|
| 465.93 | 191.33 | 0.41 |

This is because the consideration of the cohesion as an additional input variable introduces more uncertainty in the analysis.

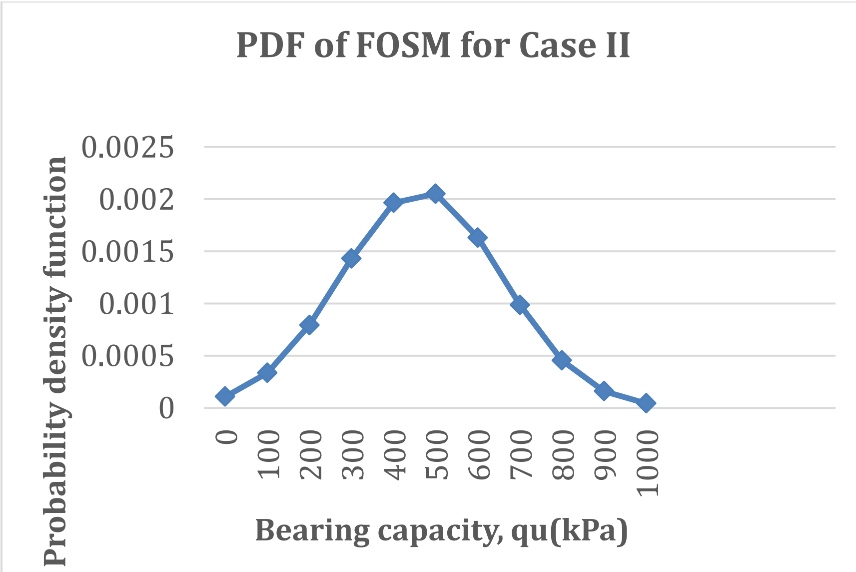



Figure 4-7 : PDF of FOSM for both soil parameters considered as uncorrelated random variables

#### *4.2.1.3 Case III: FOSM results for correlated c & ϕ soil parameters*

If the input soil variables tanϕ´ and c´ are correlated, then the variance formula will be applied taking into account the covariance. Considering a correlation coefficient of ρc´tanϕ´ = -0.7, then the resulting statistical values of the bearing capacity will be those presented in Table 4.11. When compared to case1, the mean value does not change; instead the standard deviation decreases significantly, thus reducing also the COVqu.

Table 4-11 : Statistical values of qu predicted by FOSM method for case III with ρc´tanϕ´ = -0.7 correlation

| μqu(KN/$m^2$) | σ qu(KN/$m^2$) | COVqu |
|---|---|---|
| 465.93 | 131.03 | -0.175 |

Summarizing all these observations it seems to be very important for probabilistic analysis to include a negative correlation between cohesion and friction angle in order to have less uncertainty in the final results.

Table 4-12 : Influence of ρc´tanϕ´ on the statistical values of qu from FOSM

| ρc´tanϕ´ | COV c´tanϕ´ | σ qu(KN/$m^2$) |
|---|---|---|
| -1.0 | -0.249 | 94.01 |
| -0.9 | -0.224 | 107.77 |
| -0.8 | -0.199 | 119.96 |
| -0.7 | -0.175 | 131.03 |
| -0.6 | -0.149 | 141.23 |
| -0.5 | -0.125 | 150.74 |
| -0.4 | -0.099 | 159.68 |
| -0.3 | -0.075 | 168.16 |
| -0.2 | -0.049 | 176.22 |
| -0.1 | -0.025 | 183.93 |
| 0.0 | 0.000 | 191.33 |

### 4.2.2 Second Order Second Moment (SOSM) method

FOSM method can be slightly extended for a better prediction of the bearing capacity mean value through the SOSM method. Results of this approach will be illustrated in the next section and compared to FOSM method.

#### *4.2.2.1 SOSM results for only ϕ as random variable*

Second order terms of Taylor's series expansion of the bearing capacity will be defined as follows. In this way it is possible to refine the estimate of the bearing capacity mean value, leading to:

$$\mu qu = qu(\mu \tan \phi') + (\tan \phi' - \mu \tan \phi')\frac{\boldsymbol{\partial qu}}{\boldsymbol{\partial \tan \phi'}} + \frac{\mathbf{1}}{\mathbf{2}}\,\mathrm{Var}(tan\ \phi')\,\frac{\partial^2 qu}{\partial tan^2\ \phi'} \quad \text{………….. Equation 4-11}$$

Substituting the numerical values for unit weight, surcharge, cohesion and the mean value of tanϕ´, the first derivative of the bearing capacity is the same as for FOSM method in previous section, while the second derivative is given by;

$\frac{\partial^2 qu}{\partial tan^2\, \phi'}$ = 12247.687

Then the bearing capacity means value is;

μqu = 465.93+0.5 x 0.0036 x 12247.687

μqu = 487.98 kN/$m^2$.

Var (qu) = Var($tan\phi$) $x$ ($\frac{\partial q}{\partial tan\phi'}$)$^2$ =(0.06 )$^2$ x (2880.31)$^2$

Var(qu) = 29866.27 (kN/m$^2$)$^2$

σ qu = 172.82 kN/m$^2$

The statistical values of the bearing capacity derived from SOSM application are shown in Table 4.13. Comparing these results to those of FOSM method, one sees that the second order terms have increased the mean value of the bearing capacity μqu and thus reduced the COVqu value, while the standard deviation remains constant.

Table 4-13 : Statistical values of qu from SOSM method for friction angle as random variable

| μqu(kN/$m^2$) | σ qu(kN/$m^2$) | COVqu |
|---|---|---|
| 487.98 | 172.82 | 0.354 |

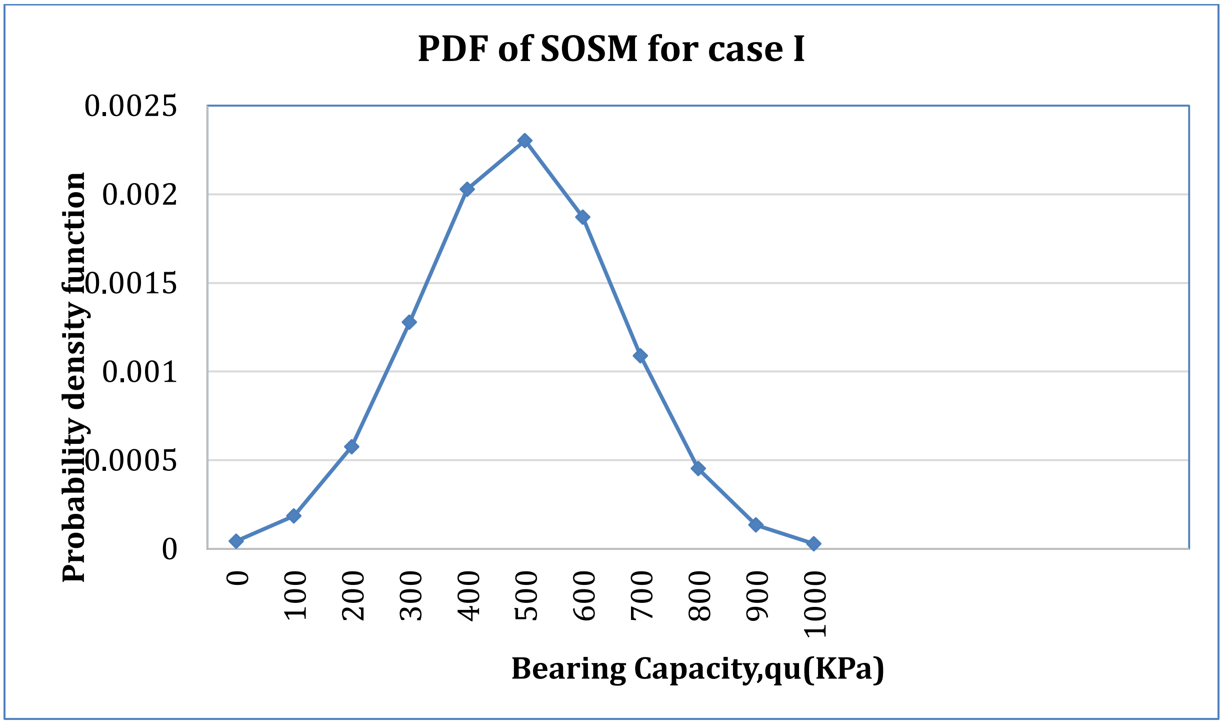



Figure 4-8 : PDF of SOSM for friction angle as random variable

*4.2.2.2 Case II: SOSM results for c and tan $\phi$ as uncorrelated random variables*

By considering the cohesion as input random variable, the mean value bearing capacity formula will include second order terms of the Taylor's series expansion, thus leading to:

$$\mu qu = qu(\mu\tan\phi', \mu c') + (\tan\phi' - \mu\tan\phi')\cdot\frac{\partial qu}{\partial \tan\phi'} + (c' - \mu c')\frac{\partial qu}{\partial c'} + \frac{1}{2}\mathrm{Var}(\tan\phi')\frac{\partial^2 qu}{\partial \tan^2\phi'} + \frac{1}{2}\mathrm{Var}(c')\frac{\partial^2 qu}{\partial c'^2} \quad \text{Equation 4-12}$$

Where derivatives are evaluated at the mean values μtanϕ´ and μc´. The first derivatives of the bearing capacity have the same values found for the FOSM method. While the second derivatives with respect to cohesion and tanϕ´. Substituting the mean values of cohesion and tanϕ´, one obtains;

$$\frac{\partial^2 qu}{\partial \tan^2\phi'} = 12247.687 \mathrm{kN}/m^2$$

$$\frac{\partial^2 qu}{\partial c'^2} = 0\ \mathrm{kN}/m^2$$

Then the bearing capacities mean value will be:

μqu = 487.98kN/$m^2$.

The corresponding statistical estimates of the bearing capacity are shown in Table 4.14.

Table 4-14 : Statistical values of qu from SOSM method for uncorrelated random variables

| μqu(kN/$m^2$) | σ qu(kN/$m^2$) | COVqu |
|---|---|---|
| 487.98 | 191.33 | 0.392 |

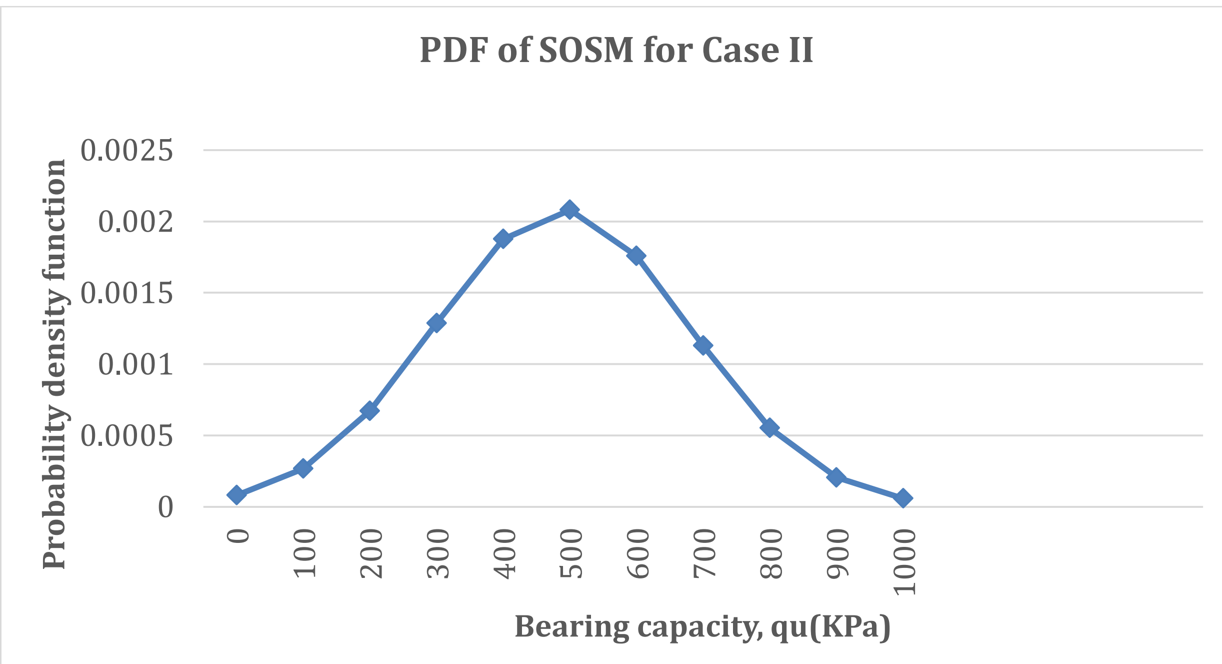



Figure 4-9 : PDF of SOSM for uncorrelated random variables

*4.2.2.3 Case III: SOSM results for correlated random variables*

Taking into account c´ and tanϕ´ as negatively correlated variables with ρc´tanϕ´ = -0.7, the mean value equation for the bearing capacity will be applied by considering the covariance, leading to μqu = 876.47 kN/$m^2$.

Varying the correlation coefficient from 0 to −1.0 causes variation of the mean value of the bearing capacity as shown in Table 4.15. On the other hand, the standard deviation values are the same of those presented in Table 4.12 from FOSM method. When compared with the FOSM results, a decrease of the correlation coefficient results in a reduction of both mean value and standard deviation of the bearing capacity. Additionally, as already seen for the FOSM application, a perfect negative correlation between cohesion and tanϕ´ strongly influences the bearing capacity variability.

Summing up all these observations, it would seem that SOSM method can refine the estimate of the bearing capacity mean value.

Table 4-15 : Influence of ρc´tanϕ´ on the statistical values of qu from SOSM

| ρc´tanϕ´ | μqu(KN/$m^2$) | σ qu(KN/$m^2$) |
|---|---|---|
| -1.0 | 438.52 | 90.70 |
| -0.9 | 443.46 | 105.19 |
| -0.8 | 448.41 | 117.91 |
| -0.7 | 453.35 | 129.39 |
| -0.6 | 458.30 | 139.93 |
| -0.5 | 463.25 | 149.82 |
| -0.4 | 468.19 | 158.92 |
| -0.3 | 473.14 | 167.61 |
| -0.2 | 478.09 | 175.87 |
| -0.1 | 483.03 | 183.76 |
| 0.0 | 487.98 | 191.33 |

### 4.2.3 The Two Point Estimate Method applied to the bearing capacity problem

*4.2.3.1 Case I: PEM results for only tanϕ as random variable*

In order to assess the bearing capacity statistical values related to given soil properties the Two Point Estimate method after ROSENBLUETH (1975) is applied. The procedure for implementing the PEM and the corresponding calculations are described step by step in next section.

**Procedure of the PEM**

1. The relationship between the dependent variable qu and the single random variable tanϕ´ will be considered.

2. The two sampling point locations for tanϕ´, which is normally distributed (νtan ϕ'= 0), have to be computed. First of all, the standard deviation units, giving locations of the sampling points to the right and to the left of the mean value, are evaluated; thus giving:

ξ 'tanϕ'+ = ξ 'tanϕ'-=1.

Then the corresponding sampling point locations can be found as follows:

tan ϕ′+ = μ tan ϕ′+ ξ tanϕ′+. σtan ϕ′ ………………………………………………………… Equation 4-13

tan ϕ′+ = 0.4452+1 x 0.0587

tan ϕ′+ = 0.5039

tan ϕ′- = μ tan ϕ′- ξ tanϕ'+. σtan ϕ′ …………………………………………………………. Equation 4-14

tan ϕ′- = 0.4452-1 x 0.0587

tan ϕ′- = 0.3865

3. The weights are determined for the two sampling points as follows:

In this case there is only one input variable, for this reason no correlation coefficient is considered in the weights formula. For tanϕ´ the weights are simply given in equation 4.15 & 4.16 and, because of the symmetry of the normal distribution, they will have the same value, i.e.

$P_{\tan\phi'+} = \xi_{\tan\phi'+} / (\xi'_{\tan\phi'+} + \xi_{\tan\phi'-})$ ………………………………………… Equation 4-15

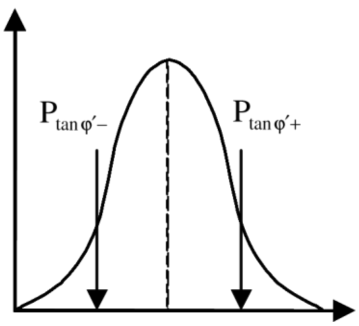
$P_{\tan\phi'+} = 1/(1+1) = 0.5$

$P_{\tan\phi'-} = \xi'_{\tan\phi'-} / (\xi_{\tan\phi'+} + \xi_{\tan\phi'-})$ ………………………………………… Equation 4-16

$P_{\tan\phi'-} = 1/(1+1) = 0.5$

4. The values of the bearing capacity are then evaluated at both sampling point locations of tanϕ´. The results are shown in Table 4.18.

f(tanφ´)




$\tan\phi'_-$ $\mu_{\tan\phi'}$ $\tan\phi'_+$

Figure 4-10 : Sampling point locations and weights for tanϕ´

´ Table 4-16 : Weights and sampling points of tanϕ´ and bearing capacity values for PEM

| P± | tan ϕ′± | qu± |
|---|---|---|
| 0.5 | 0.5039 | 813.04 |
| 0.5 | 0.3865 | 455.63 |

5. Now the first three moments of the bearing capacity can be calculated as follows:

**First Moment**

$\mu_{qu} = 0.5(813.04) + 0.5(455.63)$

$\mu_{qu} = 634.34 kPa$

**Second Moment**

$\sigma qu^2 = 0.5(813.04 - 634.34)^2 + 0.5(455.63 - 634.34)^2$

$\sigma qu^2 = 31935.48(kPa)^2$

$\sigma qu = 178.71kPa$

**Third Moment**

$\nu qu = \frac{1}{\sigma qu^3}(0.5(813.04 - 634.34)^3 + 0.5(455.63 - 634.34)^3)$

$\nu qu = 0.00008$

$\nu qu = 0.00$

The results of the bearing capacity predicted by PEM are presented in Table 4.17. In addition the skewness coefficient is nil, thus suggesting a symmetric probability density function for the bearing capacity. However, for a better definition of the shape of the bearing capacity distribution more sampling points would be needed.

Table 4-17 : Statistical values of qu predicted by PEM for case 1

| μqu(KPa) | σ qu(KPa) | COVqu | νqu |
|---|---|---|---|
| 634.34 | 178.71 | 0.282 | 0.00 |

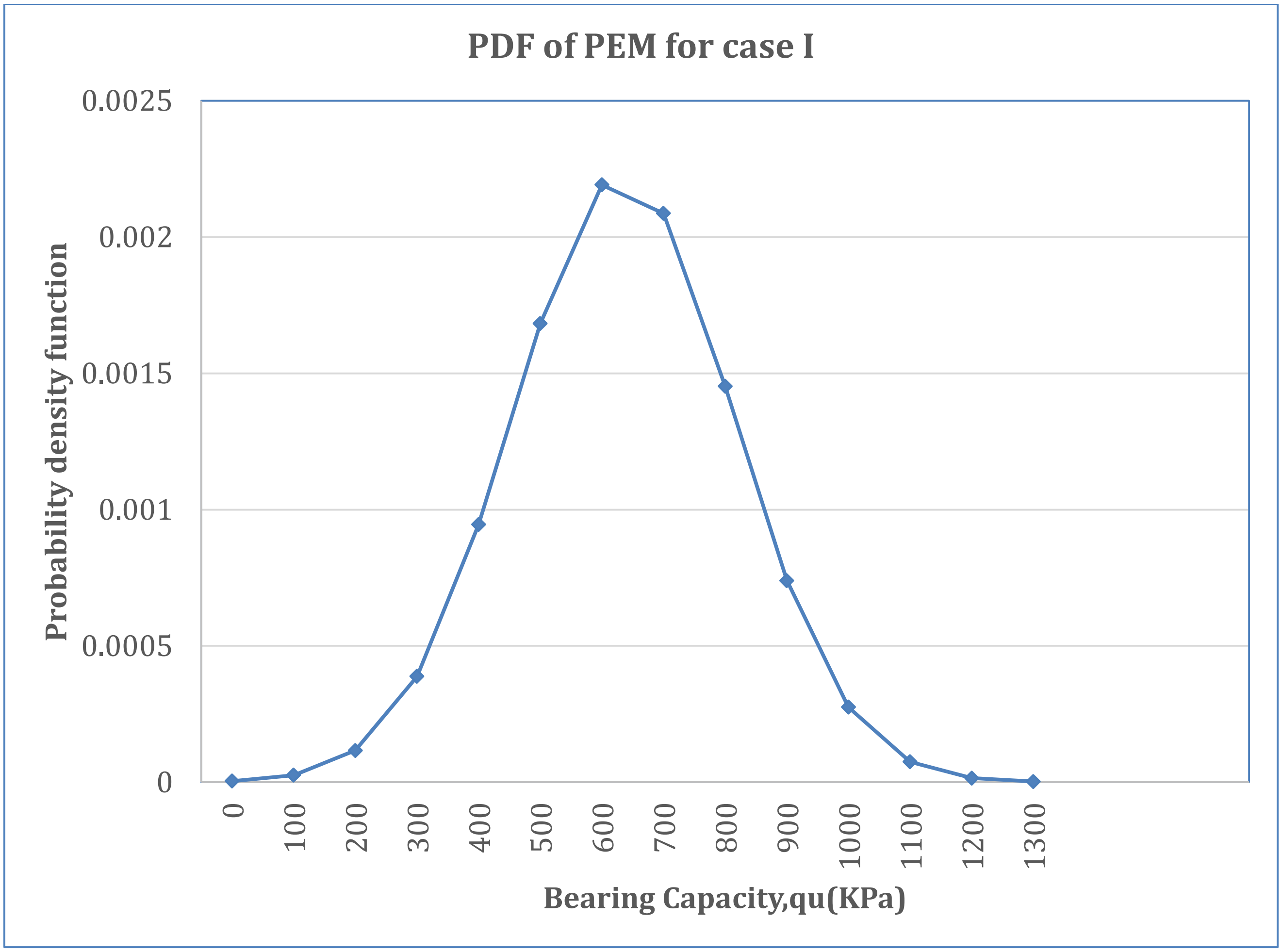



Figure 4-11 : PDF of PEM for only friction angle as random variable

*4.2.3.2 Case 2: PEM results for uncorrelated soil parameters*

For this case the number of input variables increases, thus increasing the number of PEM calculations from 2 to 4 (because n=2).

**Procedure of the PEM**

1. The relationship between the dependent variable qu and the random input variables tanϕ´ and c´ is considered.

2. Then the sampling point locations for tanϕ´ and c´ are computed. The standard deviation units will be evaluated for both soil parameters; thus leading to:

$\xi_{\tan\phi'+} = \xi_{\tan\phi'-} = 1$

$\nu c = 3\ x\ \frac{\sigma c}{\mu c} + (\frac{\sigma c}{\mu c})^2$

$\nu c = 3\ x\ \frac{4.25}{8.5} + (\frac{4.25}{8.5})^2 = 1.75$

$\xi\ c'+ = 1.75/2+(1+(1.75/2)^2)^{1/2} = 2.20$

$\xi c'- = 2.20 - 1.75 = 0.454$

Then the corresponding sampling point locations can be found as:

$\tan\phi'+ = \mu\tan\phi' + \xi\tan\phi'+ .\ \sigma\tan\phi'$

$\tan\phi'+ = 0.4452+1 \times 0.0587$

$\mathbf{\tan\phi'+ = 0.5039}$

$\tan\phi'- = \mu\tan\phi' - \xi\tan\phi'+ .\ \sigma\tan\phi'$

$\tan\phi'- = 0.4452 - 1 \times 0.0587$

$\mathbf{\tan\phi'- = 0.3865}$

$c'+ = c' + \xi\ c'+ .\sigma c'$

$c'+ = 8.5 + 2.20 \times 4.25$

**$c'+$ = 17.85kPa**

$c'- = c' - \xi\ c'- .\sigma c'$

$c'- = 8.5 - 0.454 \times 4.25$

**$c'-$ = 6.57kPa**

3. The weights Pi, giving each of the four point estimates of soil parameters considered as single random variable, are then determined as follows:

$P\ \tan\phi'+ = \xi\ '\tan\phi'+ /(\ \xi\ '\tan\phi'+ + \xi\ '\tan\phi'-)$

$P\ \tan\phi'+ = 1/\ (1+1) = 0.5$

$P\ \tan\phi'- = \xi\ '\tan\phi'- / (\xi\ '\tan\phi'+ + \xi\ '\tan\phi'-)$

$P\ \tan\phi'- = 1/(1+1) = 0.5$

$P\ c'+ = \xi\ c'+ / (\xi\ c'+ + \xi\ c'-)$

$P\ c'+ = 2.20/(2.20+0.454) = 0.8289$

$P\ c'- = \xi\ c'- / (\xi\ c'+ + \xi\ c'-)$

$P\ c'- = 0.454/2.654 = 0.1711$

Weights value:

$P+ + = P\ \tan\phi'+ .\ P\ c'+ = 0.5 \times 0.8289 = 0.4144$

$P+ - = P\ \tan\phi'+ .\ P\ c'- = 0.5 \times 0.1711 = 0.0856$

$P- + = P\ \tan\phi'- .\ P\ c'+ = 0.5 \times 0.8289 = 0.4144$

$P- - = P\ \tan\phi'- .\ P\ c'- = 0.5 \times 0.1711 = 0.0856$

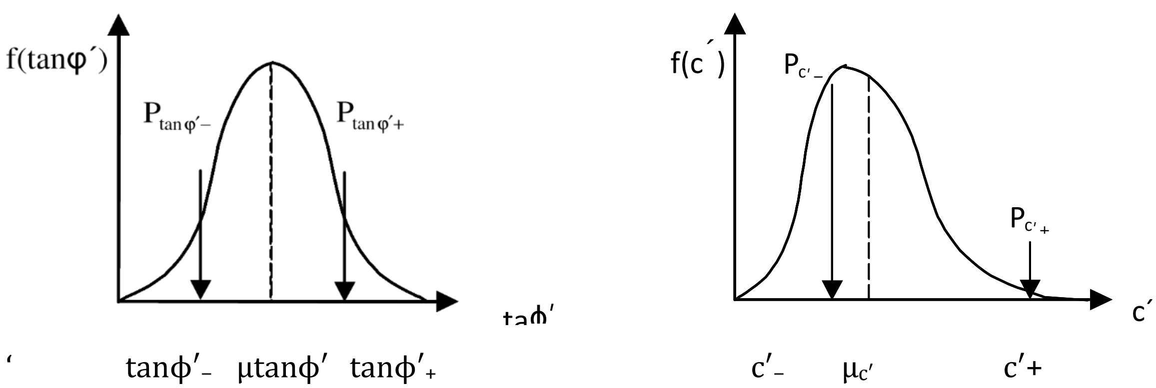



Figure 4-12 : Sampling point locations and weights of the soil parameters tanφ´ and c´

4. The dependent variable qu is evaluated at each of the points. Table 4.18 summarizes the values of the weights, the sampling points and qu for this case.

Table 4-18 : Associated weights, sampling points and bearing capacity values for tanϕ´ and c´

| P± ± | tan ϕ′± ± | c′± ± ( KPa) | qu± ± ( kPa) |
|---|---|---|---|
| 0.4144 | 0.5039 | 17.85 | 1081.947 |
| 0.0856 | 0.5039 | 6.57 | 757.534 |
| 0.4144 | 0.3865 | 17.85 | 634.678 |
| 0.0856 | 0.3865 | 6.57 | 418.666 |

5. The mean value, variance and skewness coefficient of the bearing capacity are calculated below.

**First Moment**

$\mu qu = 0.4144(1081.947) + 0.0856\,(757.534) + 0.4144\,(634.678) + 0.0856\,(418.666)$

$\mu qu = 812.05 kPa$

**Second Moment**

$\sigma qu^2 = 0.4144\,(1081.947 - \mu qu)^2 + 0.0856\,(757.534 - \mu qu)^2 + 0.4144\,(634.678 - \mu qu)^2 + 0.0856\,(418.666 - \mu qu)^2$

$\sigma qu^2 = 56725.17 (kPa)^2$

$\sigma qu = 238.17 kPa$

**Third Moment**

$\nu qu = \frac{1}{\sigma qu^3} * (0.4144\,(1081.947 - \mu qu)^3 + 0.0856\,(757.534 - \mu qu)^3 + 0.4144\,(634.678 - \mu qu)^3 + 0.0856\,(418.666 - \mu qu)^3)$

$\nu qu = \frac{1}{238.17^3} \times (0.4144\,(1081.947 - 812.05)^3 + 0.0856\,(757.534 - 812.05)^3 + 0.4144\,(634.678 - 812.05)^3 + 0.0856\,(418.666 - 812.05)^3)$

$\nu qu = 0.045$

The results of the statistical estimates of the bearing capacity predicted by PEM are presented in Table 4.19.

Comparing these results with those of Table 4.15, it can be seen that the mean value and the standard deviation increases. This also increases the skewness and the variation coefficients. The higher standard deviation is due to the consideration of the effective cohesion as input random variable, which introduces more uncertainty in the final results.

Table 4-19 : Statistical values of qu predicted by PEM with uncorrelated soil variables

| μqu(kPa) | σ qu(kPa) | COVqu | Vqu |
|---|---|---|---|
| 812.05 | 238.17 | 0.293 | 0.045 |

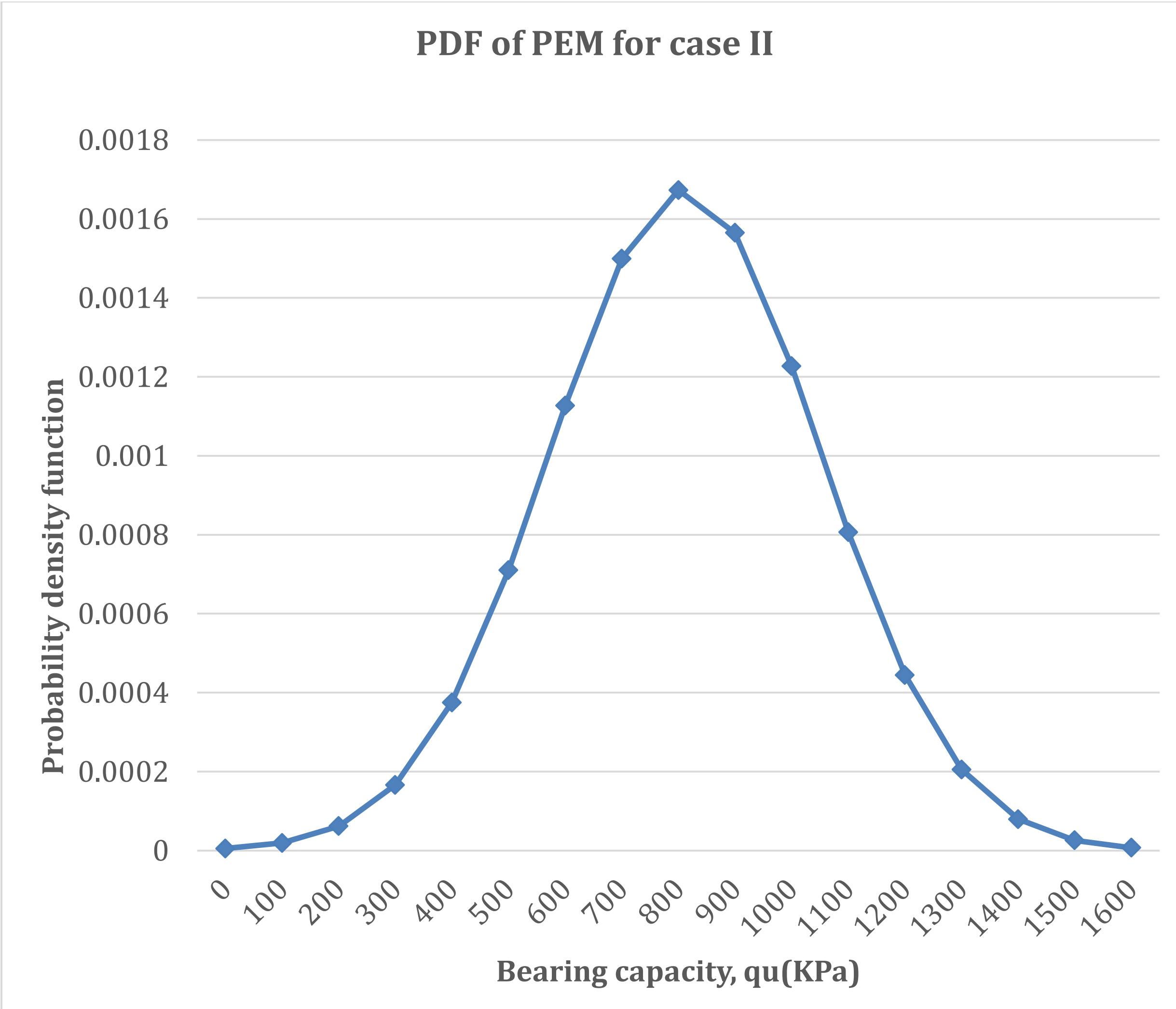



Figure 4-13 : PDF of PEM for uncorrelated random variables

*4.2.3.3 PEM results with correlated cohesion and friction angle soil parameters*

In this way it is possible to simplify calculations working with two symmetrically distributed and correlated variables. The bearing capacity statistical values are found by following the usual procedure for implementing the PEM, as shown stepwise in next section.

**Procedure of the PEM**

1. The relationship between qu and the soil variables tanϕ´ and c´ is again considered.

2. As tanϕ´ and lnc´ are both symmetrically distributed, then the standard deviation units of the soil parameters will be both equal to unity.

$\xi$ tanϕ′+ = $\xi$ tanϕ′-=1

$\xi$ lnc′+ = $\xi$inc′- =1

Then the corresponding sampling point locations can be found as:

tan ϕ′+ = μ tan ϕ′+ $\xi$ 'tanϕ′+. σtan ϕ′

tan ϕ′+ = 0.5039

tan ϕ′- = μ tan ϕ′- $\xi$ 'tanϕ′+. σtan ϕ'

tan ϕ′- = 0.3865

lnc′+ = lnc' + $\xi$ln c′+.σc′

lnc′+ = 2.14+1 x 1.4469

lnc′+ = 3.5869kPa

lnc′- = lnc′ - $\xi$ln c′-.σc′

lnc′- = 2.14 – 1 x 1.4469

lnc′- = 0.6931kPa

3. The weights Pi, giving each of the four point estimates of the soil parameters considered as single random variable, are then determined using following formulae:

P tan ϕ′+ = $\xi$ tanϕ′+ /( $\xi$ tanϕ′+ + $\xi$ tanϕ′-)

P tan ϕ′+ = 1/ 2 = 0.5

P tan ϕ′- = $\xi$ tanϕ′- / ($\xi$ tanϕ′+ + $\xi$ tanϕ′-)

P tan ϕ′- = 1/ 2= 0.5

P lnc′+ = $\xi$ lnc′+ / ($\xi$ lnc′+ + $\xi$ln c′-)

Plnc′+ = 1/2 = 0.5

Plnc′- = ξln c′- / (ξlnc′+ + ξlnc′-)

Pc′- = 1/ 2 = 0.5

Associated weights:

P+ + = P- - = P tan ϕ′+/-. P lnc′+/- . (1+ ρc´tanϕ´)

P+ - = P- + = P tan ϕ′+/-. P lnc′-/+.(1- ρc´tanϕ´)

These Formulae are used to find the associated weights. The sampling weights calculated for various correlation coefficients between 0 to –1.0 are listed in Table 4.18.

Table 4-20 : Associated weights of tanϕ´ and c´ varying the correlation coefficient

| ρc´tanϕ´ | Associated Weights | | | |
|---|---|---|---|---|
| | P+ + | P+ - | P- + | P- - |
| -1.0 | 0.000 | 0.500 | 0.500 | 0.000 |
| -0.9 | 0.025 | 0.475 | 0.475 | 0.025 |
| -0.8 | 0.050 | 0.450 | 0.450 | 0.050 |
| -0.7 | 0.075 | 0.425 | 0.425 | 0.075 |
| -0.6 | 0.100 | 0.400 | 0.400 | 0.100 |
| -0.5 | 0.125 | 0.375 | 0.375 | 0.125 |
| -0.4 | 0.150 | 0.350 | 0.350 | 0.150 |
| -0.3 | 0.175 | 0.325 | 0.325 | 0.175 |
| -0.2 | 0.200 | 0.300 | 0.300 | 0.200 |
| -0.1 | 0.225 | 0.275 | 0.275 | 0.225 |

| | | | | |
|---|---|---|---|---|
| 0.0 | 0.250 | 0.250 | 0.250 | 0.250 |

4. The values of the bearing capacity are then evaluated at each sampling point locations. To determine the bearing capacity values at each sampling point, the sampling point locations of lnc´ need to be transformed into the sampling point locations of the lognormal cohesion. This is done simply using the exponential function,

i.e.

$c'+ = e^{\ln c'+} = e^{3.5869} = 36.1219$

$c'- = e^{\ln c'-} = e^{1.6931} = 5.4363$

E.g for ρc´tanϕ´ = -0.5,

Table 4-21 : Associated weights, sampling points and bearing capacity values for tanϕ´ and lnc´

| P± ± | tan ϕ′±± | lnc′± ± | c′± ± | qu± ± |
|---|---|---|---|---|
| 0.125 | 0.5039 | 3.5869 | 36.1219 | 1607.39 |
| 0.375 | 0.5039 | 1.6931 | 5.4363 | 724.75 |
| 0.375 | 0.3865 | 3.5869 | 36.1219 | 984.55 |
| 0.125 | 0.3865 | 1.6931 | 5.4363 | 396.83 |

5. The first three moments of qu calculated below.

Where n = 2.

**First Moment**

$\mu qu = 0.125\ (1607.39) + 0.375\ (724.75) + 0.375\ (984.55) + 0.125\ (396.83)$

$\mu qu = 891.52$

**Second Moment**

$\sigma qu^2 = 0.125\ (1607.39 - \mu qu)^2 + 0.375\ (724.75 - \mu qu)^2 + 0.375\ (984.55 - \mu qu)^2 + 0.125\ (396.83 - \mu qu)^2$

$\sigma qu^2 = 0.125\ (1607.39 - 891.52)^2 + 0.375\ (724.75 - 891.52)^2 + 0.375\ (984.55 - 891.52)^2 + 0.125\ (396.83 - 891.52)^2$

$\sigma qu^2 = 108323.56 (kPa)^2$

$\sigma qu = 329.125 kPa$

**Third Moment**

$\nu qu = \frac{1}{\sigma qu^3}\, x\ 0.125\ (1607.39 - \mu qu)^3 + 0.375\ (724.75 - \mu qu)^3 + 0.375\ (984.55 - \mu qu)^3 + 0.125\ (396.83 - \mu qu)^3$

$\nu qu = \frac{1}{329.125^3}\, x\ 0.125\ (1607.39 - 891.52)^3 + 0.375\ (724.75 - 891.52)^3 + 0.375\ (984.55 - 891.52)^3 + 0.125\ (396.83 - 891.52)^3$

$\nu qu = 0.82$

The statistical values of the bearing capacity corresponding to a correlation coefficient of –0.7 are shown in Table 4.22.

Table 4-22 : Statistical values of qu predicted by PEM with ρc´tanϕ´ = -0.7

| μqu(kPa) | σ qu(KPa) | COVqu | νqu |
|---|---|---|---|
| 876.768 | 268.46 | 0.306 | 1.378 |

Varying the correlation coefficient from 0 to –1.0, the statistical values of the bearing capacity change as shown in Table 4.23. By decreasing the correlation coefficient and considering the normal variable lnc´ as input for the analysis, the mean value changes slightly for uncorrelated soil parameters to parameters with ρc´tanϕ´ = -1.0, while the standard deviation decreases significantly for the case with ρc´tanϕ´ = -1.0.

Table 4-23 : PEM statistical values of the bearing capacity for different correlation coefficients

| ρc´tanϕ´ | μqu(KPa) | σqu(KPa) | νqu |
|---|---|---|---|
| -1.0 | 854.648 | 129.9 | 0 |
| -0.9 | 862.021 | 188.09 | 1.124 |
| -0.8 | 869.394 | 231.91 | 1.134 |
| -0.7 | 876.768 | 268.46 | 1.378 |
| -0.6 | 884.141 | 300.42 | 1.186 |
| -0.5 | 898.887 | 329.13 | 1.029 |
| -0.4 | 906.261 | 355.37 | 0.899 |
| -0.3 | 913.634 | 379.66 | 0.788 |
| -0.2 | 921.007 | 402.35 | 0.690 |
| -0.1 | 928.381 | 423.71 | 0.604 |
| 0.0 | 928.381 | 443.91 | 0.526 |

It is possible to conclude that the choice of a negative correlation between soil parameters is reasonable, because the uncertainty in the probabilistic analysis is effectively reduced.

## 4.3 Comparison of FOSM, SOSM and PEM results

The scope of this section is to compare PEM results for cases I, II and III already shown above to those of FOSM, SOSM methods. FOSM and SOSM methods do not provide information about the skewness coefficient. The PEM is chosen as alternative probabilistic method to be applied to the bearing capacity problem, instead of FOSM and SOSM methods, because it requires much less computational effort and provides information about the skewness coefficient. Furthermore this approach does not require the determination and evaluation of partial derivatives of the bearing capacity formula as FOSM and SOSM, thus being more straightforward to use.

### 4.3.1 Comparison of results for case I

In Table 4.24 the statistical values of the bearing capacity for case 1 found by applying the probabilistic methods PEM, FOSM and SOSM are listed. There is slight difference between mean values. In fact the FOSM and SOSM method do not provide any skewness coefficient, while the PEM provides a skewness value. Thus the PEM is more accurate than FOSM and SOSM methods, giving the additional information about the shape of the bearing capacity distribution. Actually, the PEM skewness coefficient suggests that a normal distribution should be assumed for approximating the bearing capacity statistical values. It is important to observe that PEM requires less calculations to get the results. Thus, by applying PEM, the computational effort considerably decreases.

Table 4-24 : Comparison of PEM, FOSM and SOSM for case I

| Methods | μqu | σ qu | COVqu |
|---|---|---|---|
| FOSM | 465.93 | 172.82 | 0.37 |
| SOSM | 487.98 | 172.82 | 0.354 |
| PEM | 634.34 | 178.71 | 0.282 |

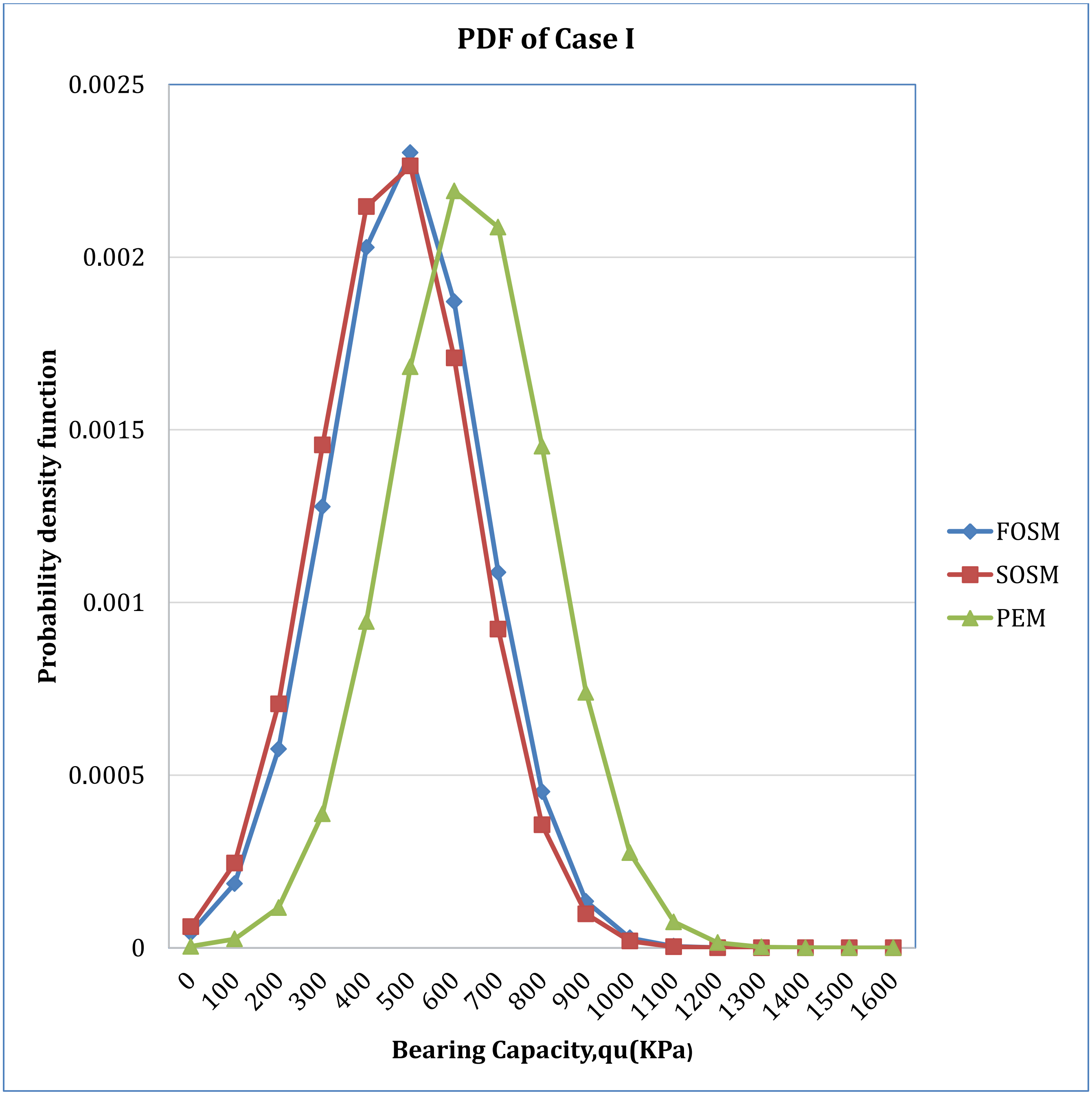



Figure 4-14 : Comparison of results for only friction angle as random variable

### 4.3.2 Comparison of results for case II with uncorrelated soil parameters

Table 4.25 summarizes the statistical values of the bearing capacity previously evaluated by PEM, FOSM and SOSM methods for case II considering uncorrelated soil parameters. Comparing these results, it can be seen that mean values and standard deviations of FOSM and SOSM are quite similar, while there is a difference from PEM. . It is important to notice that, for this case, PEM requires only four calculations to get the results of Table 4.25.

Table 4-25 : Case II Uncorrelated random variables comparison

| Methods | μqu | σ qu | COVqu |
|---|---|---|---|
| FOSM | 465.93 | 191.33 | 0.41 |
| SOSM | 487.98 | 191.33 | 0.392 |
| PEM | 812.05 | 238.17 | 0.293 |

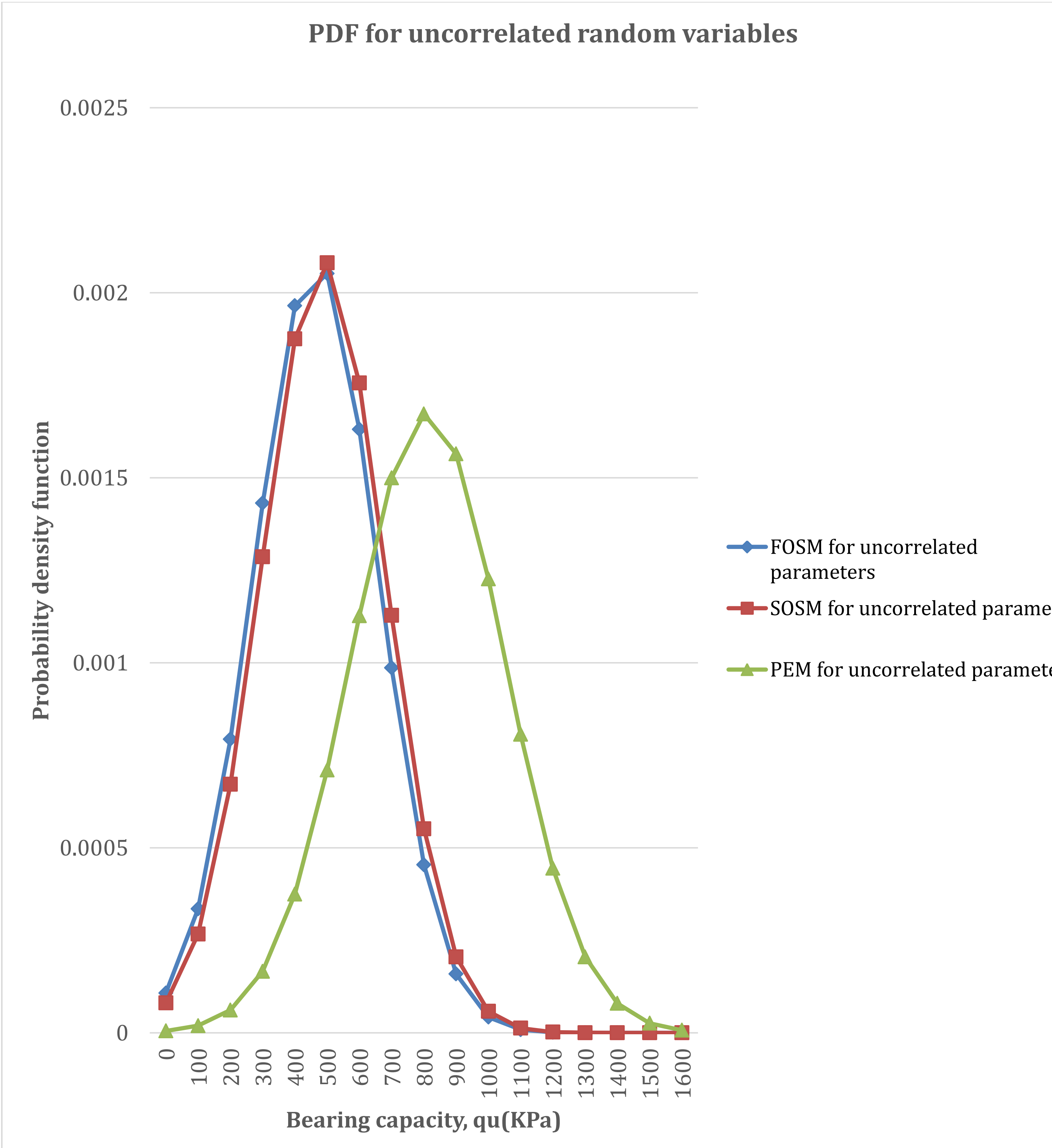



Figure 4-15 : Comparison of results for only friction angle as random variable

### 4.3.3 Comparison of results for case III with correlated soil parameters

The statistical values of the bearing capacity found by applying PEM, FOSM, SOSM methods to case III taking into account a negative correlations of ρc´tanϕ´ are listed in Table 4.26. In fact FOSM and SOSM methods do not provide any value, while PEM provides the skewness coefficients. In order to apply Christian's formula to evaluate the associated weights, the normal variable lnc´ needs to be considered, thus influencing the correlation coefficient between cohesion and friction angle, because of the different mean value and standard deviation of lnc´ and c´.

Table 4-26 : Case III correlated random variable comparison for different reliability based design methods

| ρc´tanϕ´ | μqu | | | σ qu | | |
|---|---|---|---|---|---|---|
| | FOSM | SOSM | PEM | FOSM | SOSM | PEM |
| -1.0 | 465.93 | 438.52 | 854.65 | 94.01 | 90.70 | 129.9 |
| -0.9 | 465.93 | 443.46 | 862.02 | 107.77 | 105.19 | 188.09 |
| -0.8 | 465.93 | 448.41 | 869.39 | 119.96 | 117.91 | 231.90 |
| -0.7 | 465.93 | 453.35 | 876.77 | 131.03 | 129.39 | 268.46 |
| -0.6 | 465.93 | 458.30 | 884.14 | 141.23 | 139.93 | 300.42 |
| -0.5 | 465.93 | 463.25 | 898.89 | 150.74 | 149.82 | 329.13 |
| -0.4 | 465.93 | 468.19 | 906.26 | 159.68 | 158.92 | 355.37 |
| -0.3 | 465.93 | 473.14 | 913.63 | 168.16 | 167.61 | 379.66 |
| -0.2 | 465.93 | 478.09 | 921.01 | 176.22 | 175.87 | 402.35 |
| -0.1 | 465.93 | 483.03 | 928.38 | 183.93 | 183.76 | 423.71 |
| 0.0 | 465.93 | 487.98 | 928.38 | 191.33 | 191.33 | 443.91 |

## 4.4 Comparison of deterministic and reliability based analysis approaches

It is easily observed from table 4.27 that for mentioned cases ultimate bearing capacity result of reliability based design approaches is somehow in the range of deterministic methods results.

Table 4-27 : Comparision of ultimate bearing capacity results for both deterministic and reliability based design methods

| **Type of Method** | **Method of Equation** | **Ultimate Bearing Capacity Value (kPa)** | | |
|---|---|---|---|---|
| | | **Case I** | **Case II** | **Case III at** $\rho c'\tan\phi' = -1.0$ |
| Deterministic methods | Terzaghi equation | 595.58 | 595.58 | 595.58 |
| | Meyerhof | 508.09 | 508.09 | 508.09 |
| | Hansen | 580.04 | 580.04 | 580.04 |
| | Vesic | 644.70 | 644.70 | 644.70 |
| | ES EN 1997 -2015 | 437.67 | 437.67 | 437.67 |
| Reliability based design methods | FOSM | 465.93 | 465.93 | 465.93 |
| | SOSM | 487.98 | 487.98 | 438.52 |
| | PEM | 634.34 | 812.05 | 854.65 |

# Part II : Design of isolated spread footing based on selected bearing capacity results.

## 4.5 Design based on deterministic approach bearing capacity result

Take Vesic bearing capacity result from deterministic methods i.e. Qult =644.70kPa and Qall = 214.9kPa.

- **Geometrical size of the footing**

A = B x L = 2.5m x 3m = 7.5m $^2$

- **Structural design of the footing**

For C25 & S460

fcd = 11.33MPa.

fctd =fbd = 1.03MPa

fyd = 400MPa

ρmin = $\frac{0.5}{fy} = \frac{0.5}{460}$ = 0.0010869

ρx = ρy = 0.0002376 < ρmin

Therefore, take ρe = ρmin = **0.0010869**

Equations and detail calculations are given in appendix F.

- **Geometrical size of the footing**

A = B x L = 2.5m x 3m = 7.5m $^2$

- **Structural design of the footing**

For C25 & S460

fcd = 11.33MPa.

fctd =fbd = 1.03MPa

fyd = 400MPa

$\rho min = \frac{0.5}{fy} = \frac{0.5}{460} = 0.0010869$

$\rho x = \rho y = 0.0002376 < \rho min$

Therefore, take $\rho e = \rho min =$ **0.0010869**

Equations and detail calculations are given in appendix F.

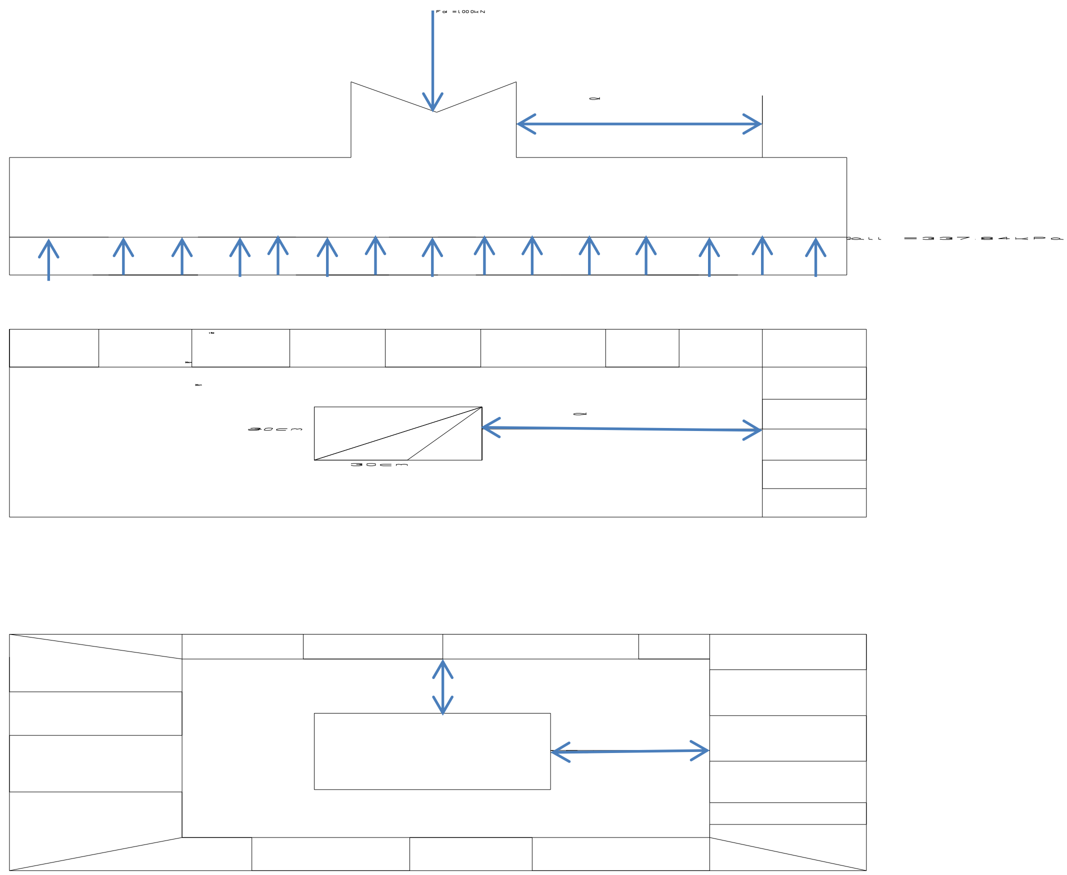


Figure 4-16 : Wide beam and punching shear cross section of footing

**I) Wide beam shear**

$k1 = 1+50\rho \leq 2$

$k1 = 1+ 50 \times 0.0010869 = 1.05$

$k2 = 1.6\text{-}d \geq 1$

take $k2 = 1$

a) Shear causing force along x-axis:

Vsd = (pressure)(shaded area)

$= \text{Allowable bearing capacity x } (\frac{L}{2} - \frac{Column\ size}{2} - d) \text{ x } B$

$= 214.9(\frac{3}{2} - \frac{0.3}{2} - d) \text{ x } 2.5$

$= 214.9(1.5 - 0.15 - d) \text{ x } 2.5$

Vsd = 725.28 – 537.25d

Concrete resistance:

Vrd =0.25 x fctd x k1 x k2 x B x d

Vrd =0.25 x 1031 x 1.05 x 1 x 2.5 x d

Vrd = 676.59d

Then:

676.59d = 725.28 – 537.25d

676.69d+537.25d = 725.28

1214.94d = 725.28

d = 0.60m =**60cm**

b) Shear causing force along Y axis

Vsd = (Pressure)*(Shaded area)

$= \text{Allowable bearing capacity x } (\frac{B}{2} - \frac{Column\ size}{2} - d) \text{ x } L$

$= 214.9(\frac{2.5}{2} - \frac{0.3}{2} - d) \text{ x } 3$

$= 214.9(1.25 - 0.15 - d) \text{ x } 3$

Vsd = 709.17 – 644.7d

Concrete resistance:

$Vrd = 0.25 \times fctd \times k1 \times k2 \times L \times d$

$Vrd = 0.25 \times 1031 \times 1.05 \times 1 \times 3 \times d$

$Vrd = 811.91d$

Then:

$811.91d = 709.17 - 644.7d$

$811.91d + 644.7d = 709.17$

$1456.61d = 709.17$

$d = 0.4869m =$ **48.69cm**

**II) Punching shear**

Shear causing force

= (Pressure) (Shaded area)

= Allowable bearing capacity x (B x L – $(3d + 0.3)^2$)

$= 214.9(2.5 \times 3 - (3d + 0.3)^2)$

$= 214.9\,(7.5 - (9d^2 + 1.8d + 0.09))$

$Vsd = -1934.1d^2 - 386.82d + 1592.41$

Concrete shear resistance

= (unit shear resistance)(shear area)

$= 270.64\,(4 \times (3d+0.3) \times d$

$Vrd = 3247.68d^2 + 324.768d$

Then:

$-1934.1d^2 - 386.82d + 1592.41 = 3247.68d^2 + 324.768d$

$-1934.1d^2 - 3247.68d^2 - 386.82d - 324.768d + 1592.41 = 0$

Using binomial equation;

d = 0.4899m = **48.99cm**

Therefore, take **d= 60cm**

Depth of the footing will be;

D = d+concrete cover + diameter of reinforcement

D = d+c+ϕ

D = 60 + 5 + 1.4 = 66.4cm =**67cm**

Consider depth of footing 85cm and d =67-5-1.4 =60.6cm

- **Reinforcement**

Since moment in both directions is the same reinforcement along both directions is also the same.

$$\rho x = \rho y = [1 - (1 - \frac{2M}{fcd \; x \; b \; x \; d^{2}})^{1/2}] \frac{fcd}{\text{fyd}}$$

$$\rho x = \rho y = [1 - (1 - \frac{2 \; x \; 70}{11.33 \; x \; 10^{3} \; x \; 1 \; x \; 0.606^{2}})^{1/2}] \frac{11.33}{400}$$

ρx = ρy = 0.0004806 < ρmin

Therefore, take ρe = ρmin = **0.0010869**

As = ρmin x b x d

= 0.0010869 x 1000 x 606

= 658.66 $mm^2$

$$S = \frac{a \; x \; b}{As}$$

$$= \frac{\pi \; x \; 7^{2} \; x \; 1000}{658.66} = 233.71\text{mm}$$

**Use ϕ14 c/c 230**

- **Development length**

$$Ld = \frac{\phi\ x\ fyd}{4fbd}$$

$$= \frac{14*400}{4*1.0315} = \mathbf{1357.25mm}$$

**Lavailable =** $\frac{1}{2}$(B - Column dimension) - concrete cover + D - 2c

$$= \frac{1}{2}(2500 - 300) - 50 + 670 - 2 \times 50$$

= **1620mm > Ld** Ok!

Therefore, deterministic approach design method gives total depth of footing 67cm and reinforcement ϕ14 c/c 230. Based on this reinforcement detail for shorter direction is 12ϕ14 c/c 180 and for longer direction 14ϕ14 c/c 180. It should be noticed that this detail shows only one face (either top or bottom).

## 4.6 Design based on PEM bearing capacity result among reliability based design methods

Deterministic method of design considers all uncertainty in the applied loads and ultimate geotechnical or structural components capacity in total factor of safety while reliability based design approach incorporates variable partial safety factors for load and resistance. In reliability based design approaches Load and Resistance Factor Design (LRFD) method will be used for design purpose.

For the Strength Limit States: From equation 3.31

$$R_r = \varphi R_n \geq \sum \eta_i \gamma_i Q_i$$

Relative to bearing capacity and sliding of a spread footing, the suitability of a spread footing with respect to the geotechnical resistance can written as:

$$Q_R = \varphi Q_{ult} \geq \sum \eta_i \gamma_i Q_i$$ ………………………………………..…………….. Equation 4-17

Where;

$Q_R$ = Factored geotechnical resistance of a spread footing

$\varphi$ = Resistance factor

$Q_{ult}$ = Ultimate geotechnical resistance of a spread footing

$\sum \eta_i \gamma_i Q_i$ = Factored load effect

Bias means difference between what is predicted and what is measured. The bias factor, λ, of Meyerhof's SPT method is defined as the ratio of the measured resistance to the predicted resistance.

$$\lambda = R_m / R_n \quad \text{..........Equation 4-18}$$

where:

$R_m$ = Measured nominal resistance

$R_n$ = Predicted nominal resistance

The data base of load tests should be large enough and should contain high quality data, so that the statistics derived from the data base will be representative of the loads and prediction practice.

Now check and compare using reliable based design methods. For this purpose use PEM method result of correlated random variables for $\rho c'\tan\phi'$ =-1.0 i.e.

$$Q_{ult} = 854.65\text{kPa}$$

$$Q_{all} = \phi\, Q_{ult}$$

**Step 1: Total unfactored loads:**

Dead load (Q) = 666.67kN

Live Load (Q) = 333.33 kN

$Q = Q_D + Q_L$

Q= (666.67 +333.33)kN = 1000kN

**Step 2: Load factors and factored loads calculation:**

Load factors are given in FHWA manual as;

$\gamma_{DL} = 1.25$

$\gamma_{QL} = 1.75$

$\eta_i = 1.0$ for typical structure.

Therefore, Total factored load effects based on strength limit state I can be calculated as follows;

$\sum \eta_i \gamma_i Q_i = \eta_i (\gamma_{DL} Q_{DL} + \gamma_{QL} Q_{QL})$

$\sum \eta_i \gamma_i Q_i = 1.0((1.25 \times 666.67) + (1.75 \times 333.33))$

$\sum \eta_i \gamma_i Q_i = 1416.67$ kN

**Step 3: Selection of target reliability index with estimation of lowest probability failure**

FHWA manual for spread footing recommends 3.5 as most reliable value of target reliability index. This value i.e. $\beta = 3.5$ is considered here.

**Step 4: Calculation of resistance factor**

Table 4-28 : Load statistics and factors used by Paikowsky et,al. (2004).

| Load type | Bias | Coefficient of variation | Load factors used |
|---|---|---|---|
| Dead load | $\lambda_{DL}$=1.08 | $COV_{DL}$=0.1 | $\gamma_{DL} = 1.25$ |
| Live load | $\lambda_{QL}$=1.15 | $COV_{QL}$=0.2 | $\gamma_{QL} = 1.75$ |

By considering above values mentioned in table 4.26 and selected target reliability index resistance factor is calculated as follows:

Take resistance factor formula mentioned before i.e.

$$\varphi = \frac{\lambda_R (\gamma_D Q_D/Q_L + \gamma_Q)\sqrt{(1+COV^2 Q_D + COV^2 Q_L)/(1+COV^2_R)}}{(\lambda_{QD} Q_D/Q_L + \lambda_{QL})\exp\{\beta_T \sqrt{\ln[(1+COV^2_R)(1+COV^2 Q_D + COV^2 Q_L)]}\}}$$

$\lambda_R = 1.3$

$COV_R = (0.34+0.44/L)^{0.5} = 0.69$

Substituting all values gives:

Calculated φ for reliability target value = 0.15

φ from FHWA for mentioned type of soil is given 0.45.

Here by considering target reliability value and manual recommended values, average value of φ is considered 0.30.

Qall **=** ϕ Qult

=0.30 x 854.65

Qall **=** 256.39kPa

- **Geometrical size of the footing**

A = B * L = 2.5m *3m = 7.5m $^2$

- **Structural design of the footing**

For C25

fcd = 11.33MPa.

fctd =fbd = 1.03MPa

For S460

fyd = 400MPa

ρmin = 0.0010869

ρx = ρy = 0.0002376 < ρmin

Therefore, take ρe = ρmin = **0.0010869**

**I) Wide beam shear**

k1 =1.05

k2 =1

Shear causing force along x-axis:

Vsd = (pressure)(shaded area)

Vsd = 256.39 (1.5-0.15-d)x2.5

Vsd = 865.32 – 640.98d

Concrete resistance:

Vrd =0.25*fctd*k1*k2*B*d

Vrd = 676.59d

Then:

676.59d = 865.32 – 640.98d

**d = 0.656m =65.6cm**

Shear causing force along Y axis

Vsd= (Pressure) (Shaded area)

= Allowable bearing capacity ($\frac{B}{2}$ - $\frac{Column\ size}{2}$ –d) x L

= 256.39 ($\frac{2.5}{2}$ - $\frac{0.3}{2}$ –d) x 3

= 256.39 (1.25- 0.15 –d) x 3

Vsd = 846.09 – 769.17d

Concrete resistance:

Vrd = 811.91d

Then: 811.91d = 846.09 – 769.17d

$d = 0.5351m =$ **53.51cm**

**II) Punching shear**

Shear causing force

= (Pressure) (Shaded area)

= Allowable bearing capacity $(B \times L - (3d + 0.3)^2)$

$= 256.39\ (2.5x3 - (3d + 0.3)^2)$

$= 256.39\ (7.5 - (9d^2 + 1.8d + 0.09))$

$Vsd = -2307.51d^2 - 461.50d + 1899.85$

Concrete shear resistance

= (unit shear resistance)(shear area)

$= 270.64 \times (4\ (3d+0.3) \times d)$

$Vrd = 3247.68d^2 + 324.77d$

Then:

$-2307.51d^2 - 461.50d + 1899.85 = 3247.68d^2 + 324.77d$

$-5555.19d^2 - 786.27d + 1899.85 = 0$

Using binomial equation;

$d = 0.5183m =$ **51.83cm**

Therefore, take **d= 65.6cm**

Depth of the footing will be;

D = d+concrete cover + diameter of reinforcement

$D = d + c + \phi$

$D = 65.6 + 5 + 1.4 =$ **72.0cm**

**Reinforcement**

Since moment in both directions is the same reinforcement along both directions is also the same.

$$\rho x = \rho y = [1 - (1 - \frac{2M}{fcd\ x\ b\ x\ d^{\ 2}})^{\ 1/2}\ ]\ \frac{fcd}{\text{fyd}}$$

$$\rho x = \rho y = [1 - (1 - \frac{2x70}{11.33\ x\ 10^{\ 3}\ x\ 1\ x\ 0.656^{\ 2}})^{\ 1/2}\ ]\ \frac{11.33}{400}$$

$\rho x = \rho y = 0.0004096 < \rho min$

Therefore, take $\rho e = \rho min$ = **0.0010869**

As = $\rho$min x b x d

= 0.0010869 x 1000 x 656

= 713.01 $mm^2$

$$S = \frac{a\ x\ b}{As}$$

$$= \frac{\pi\ x\ 7^{\ 2}\ x\ 1000}{713.01} = 215.89mm$$

**Use ϕ14 c/c 215**

- **Development length**

$$Ld = \frac{\phi\ x\ fyd}{4fbd}$$

$$= \frac{14\ x\ 400}{4\ x\ 1.0315} = \mathbf{1357.25mm}$$

**Lavailable =** $\frac{1}{2}$(B - Column dimension) - concrete cover + D - 2c

= $\frac{1}{2}$(2500 – 300) - 50 + 720 – 2 x 50

= **1670mm > Ld** Ok!

According to reliability based design it is obtained that total depth of footing is 72 cm and reinforcement spacing is ϕ14 c/c 215. Therefore use 13ϕ14 c/c 170 for shorter direction and 15ϕ14 c/c 160 for longer direction. It should be noticed that this detail shows only one face (either top or bottom).

Table 4-29 : Comparision of deterministic approach and reliability based design method spread footing design case

| Design Method | Qult (kPa) | FS or ϕ (dim.) | Qall or Qr (kPa) | d (cm) | S (mm) | No. of bars-BF (Shorter length ) | No. of bars-BF (Longer length) | Weigt of Reinf. (kg) | Cost analysis (Birr) (000) |
|---|---|---|---|---|---|---|---|---|---|
| Determinstic | 644.70 | 3 | 214.9 | 60.6 | 230 | 24 | 28 | 243.78 | 10.967 |
| RBD | 854.65 | 0.30 | 256.39 | 65.6 | 215 | 26 | 30 | 262.44 | 11.808 |

NB: From above table BF means both faces and total weight is calculated with multiplication of total bar length, number of bars and weight conversion factor of ϕ14.

It is clearly observed that in the above comparision, RBD design approach rquires higher number of reinforcements. Based on economic comparision result RBD method costs 7.67% higher than deterministic approach.

## Part II

**ANALYSIS AND DESIGN OF PILE FOUNDATION**

**Laboratory test results**

Each test purpose, significance, equupments and procedure discussed at the beginning of this chapter. Here test result for pile foundation purpose has illustrated**.**

**Location:** Arat kilo, Addis Ababa.

**Project:** G+7 Complex Building.

Table 4-30 : Laboratory test results for pile foundation input parameters

| **No.** | **Parameter** | **Test methods** | **Parameter's value** |
|---|---|---|---|
| 1 | Direct Shear<br>C (kPa)<br>Φ (Degree) | ASTM D 3080 | <br>26<br>22 |
| 2 | Specific gravity | ASTM D 854 | 2.46 |
| 3 | Bulk unit weight(gm/cc) | ASTM D 7263 | 2.53 |
| 4 | Moisture content (%) | ASTM D 2216 | 21.48 |

### 4.7 Analysis of pile resistance capacity using deterministic methods

Consider circular concrete pile of diameter 0.40m and 18m long.

Take unfactored loads DL 1300kN and LL =450 kN

$\alpha = 0.98$

DL +LL = 1300+450 =1750kN

I – Individual pile failure

$Ap = \pi r^2 = \pi \times 0.2^2 = 0.126m^2$

$As = \pi \times d \times L = \pi \times 0.4 \times 18 = 22.62m^2$

$Qu = 9Ap \times cu + \alpha \times cu \times As$

$Qu = 9 \times 0.126 \times 26 + 0.98 \times 26 \times 22.62$

$Qu = 29.484 + 576.36$

$Qu = 605.84kN$

$Qall = \frac{Qult}{FS}$

$Qall = \frac{605.84}{3.5} = 173.09$

## 4.8 Analysis of pile foundation resistance capacity using reliability based design methods

The reliability based design method includes uncertain variables such as variable load, effective friction angle of the soil and length of pile. The characteristic value for a design load is defined as the load magnitude that corresponds to 5% or 2% probability of exceedence (i.e. an upper 95% or 98% fractile of its probability distribution) (European Committee for Standardization 2002).

Step 1: **Total unfactored loads**:

Dead load (Q) = 1300kN

Live Load (Q) = 450kN

$Q = Q_D + Q_L$

Q= (1300 +450)kN = 1750kN

Step 2: **Load factors and factored loads calculation**:

Load factors are given in FHWA manual as;

$\gamma_{DL} = 1.25$

$\gamma_{QL} = 1.75$

$\eta_i$ = 1.0 for typical structure.

Therefore, Total factored load effects based on strength limit state I can be calculated as follows;

$\sum \eta_i \gamma_i Q_i = \eta_i (\gamma_{DL} Q_{DL} + \gamma_{QL} Q_{QL})$

$\sum \eta_i \gamma_i Q_i = 1.0((1.25 \times 1300) + (1.75 \times 450))$

$\sum \eta_i \gamma_i Q_i = 2412.5kN$

Step 3: Estimate Axial Capacity of Single Pile

I – Individual pile failure

$Ap = \pi r^2 = \pi \times 0.2^2 = 0.126m^2$

$As = \pi \times d \times L = \pi \times 0.4 \times 18 = 22.62m^2$

$qp = 9 \times 26 = 234kPa$

$qs = \alpha \times cu = 0.98 \times 26 = 25.48kPa$

$Qp = qp \times Ap = 234 \times 0.126 = 29.484$ kN

$Qs = qs \times As = 25.48 \times 22.62 = 576.36kN$

$Qu = Qp + Qs = 29.484 + 576.36 = 605.844kN$

The factored axial resistance of a single pile is:

$Qr = \varphi Qult = \varphi qpQp + \varphi qsQs$

From FHWA LRBD related ristance factors are:

$\varphi qp = 0.45$

$\varphi qs = 0.45$

The factored bearing resistance is then:

$Qr = \varphi qpQp + \varphi qsQs =$ 0.45 x 29.484 kN+0.45 x 576.36kN = **272.63kN**

## 4.9 Design of pile foundation using deterministic approaches

No. of piles $=\frac{\text{Qult}}{Qall} = \frac{1750}{173.09} = 11$piles

Take 4 x 4 piles with a spacing, S = 4 x 0.4 =1.6m

$\theta = \tan^{-1}(\frac{D}{S})$

$\theta = \tan^{-1}(\frac{.4}{1.6}) = 14.04$

m =n= 4

$\eta = 1- \frac{\theta}{90}[\frac{m(n-1)+n(m-1)}{mn}]$

$\eta = 1- \frac{14.04}{90}[\frac{4(4-1)+4(4-1)}{4 \, x \, 4}]$

$\eta = 0.766$

Qga = No. of piles x η x Qall

Qga = 16 x 0.766 x 173.09

Qga =2121.39kN > 1750kN

**II Block failure**

L =B =3S + d = 3 x 1.6+0.4 =5.2m

Ag = 5.2 x 5.2 = 27.04m $^{2}$

Pg = 4 x 5.2 = 20.8m $^{2}$

Qug = 9 x Ag x cu+cu x Pg x L

Qug = 9 x 27.04 x 26+26 x 20.8 x 18 =16061.76kN

Qall $= \frac{\text{Qug}}{FS} = \frac{16061.76}{3.5} = 4589.07 > 1750$kN ok!!!

Therefore, 16 piles are sufficient for given loading and soil property conditions.

## 4.10 Design of pile foundation using reliability based design methods

Determine no. of piles required, spacing and check load carrying capacity of piles.

N = ∑ηi γi Qi/Qr=2412.5kN/272.63kN

N = 9

Take 3 x 4 piles with a spacing, S = 4 x 0.4 =1.6m

$\theta = \tan^{-1}(\frac{D}{S})$

$\theta = \tan^{-1}(\frac{.4}{1.6}) = 14.04$

m = 3 and n =4

$\eta = 1 - \frac{\theta}{90}[\frac{m(n-1)+n(m-1)}{mn}]$

$\eta = 1 - \frac{14.04}{90}[\frac{3(4-1)+4(3-1)}{3 \, x \, 4}]$

η = 0.779

Qga = No. of piles x η x Qall

Qga = 12 x 0.779 x 210.08

Qga =1963.82kN > 1546.91kN

**II Block failure**

L =B =3S + d = 3 x 1.6+0.4 =5.2m

Ag = 5.2 x 5.2 = 27.04m $^2$

Pg = 4 x 5.2 = 20.8m $^2$

Qug = Qpg + Qsg

Qug = qpg x Apg +qsg x Asg

Qug = 9 x Ag x cu+cu x Pg x L

Qug = 9 x 27.04 x 26+26 x 20.8 x 18=16061.76kN

Qrg =φQug = φqpQpg+φqsQsg

Qrg= 0.45 x 16061.76kN = 7227.79 > 1546.91kN ok!!!

Here, it is observed that 12 piles satisfy requirements of load carrying capacity of piles.

Table 4-31 : Comparision of deterministic and reliability based design of pile foundation.

| Method | Qult (kN) | FS or φ | Qall or Qr | No. of Piles | Qug | Qallg or Qrg |
|---|---|---|---|---|---|---|
| Determinstic | 605.84 | 3.5 | 173.09 | 16 | 16061 | 4589 |
| RBD | 605.84 | 0.45 | 272.63 | 12 | 16061 | 7227 |

Cost analysis with respect to number of piles in the above data shows using reliability based design methods for pile foundation is economical. Determinstic method calculation gives higher number of piles than reliability based design methods.

# CHAPTER FIVE

# 5 CONCLUSION AND RECOMMENDATIONS

## 5.1 Conclusions with respect to deterministic bearing capacity analysis methods

Determinstic bearing capacity analysis methods such as Terzaghi's, Meyerhof's, Hansen's and Vesic's equation are used in chapter four for given soil sample calculations. Terzaghi's equation has some limitation especially during consideration of inclination, non linear and non homogenous soil behavior cases. The latter three methods take consideration of Terzaghi's equation limitations. The bearing capacity results of all these methods summarized in tabular form. As observed from tabular comparision data the maximum ultimate bearing capacity is Vesic's bearing capacity result and minimum value is recorded from Euro code calculation. Most of other mehods are slightly approached. Vesic's equation is almost same as Hansen's equation which considers all factors which will affect bearing capacity result. The difference between two equations is $N\gamma$ formula. There is 9.06% variation of bearing capacity result between two methods due to difference of this bearing capacity factor. Likewise there are also variations with other equations but for design purpose of footing Vesic bearing capacity result is taken since it is latest version of other three equations and it considers more conditions during calculation of bearing capacity.

## 5.2 Conclusions with respect to reliability based design methods

In chapter 4 well-known probabilistic methods were applied to study the given soil sample on the bearing capacity of a strip footing. The moment methods FOSM and SOSM were considered to calculate bearing capacity. These methods involved a limited amount of calculations. In spite of their results in terms of mean value and standard deviation, these methods did not provide any skewness coefficient of the bearing capacity. Thus, no information was given about the shape of the probability density function of the bearing capacity. In addition, these approaches are generally accurate only for linear functions, thus the accuracy diminishes as the non-linearity of a function increases. Since the bearing capacity is a highly non-linear function of the effective friction angle, then some inaccuracies were introduced in the final results of FOSM and SOSM methods. Consequently, it was necessary to choose another probabilistic alternative, which could be able to overcome the drawbacks of FOSM and SOSM methods, providing the skewness coefficient together with the other statistical estimates, and also with less computational efforts. This method is two point estimation method (PEM).

PEM method has different advantages over other reliability based methods as mentioned in chapter 4 comparision cases. Less no. of equations it uses, its straight forwardness, its simplicity during calculations, its result accuracy makes PEM to be the most accurate and reliable probabilistic method for bearing capacity calculations among observed probabilistic methods. Although PEM was shown to be a simple and powerful technique for probabilistic analysis, it has some limitations and one should take care about these issues. As some familiar users of this method mentioned;

i) In order to reduce the error in the PEM results, the variation coefficient of the input random variables should not be large,

ii) when multiple input random variables are considered, the skewness coefficient can only be reliably calculated by applying formula of the PEM after ROSENBLUETH if the variables are uncorrelated and if the performance function is linear, which is not the case for the bearing capacity formula,

iii) To cope up with the problem of condition ii), use CHRISTIAN et al. (1999) formula to evaluate the sampling point weights of the PEM for correlated input parameters.

- **Conclusions with respect to the correlation between the soil parameters c´ and tanϕ´**

It is clearly observed that in probabilistic calculations different correlations between cohesion and friction angle were taken into account for bearing capacity analysis cases. When a negative correlation is considered lower bearing capacity variability and much lower failure probabilities (conversely, much higher reliability indices) are observed.Thus considering negative correlation during probabilistic calculation of bearing capacity affecting the final results of the probabilistic and reliability analyses significantly. It can be concluded that the choice of a negative correlation between soil parameters is reasonable, because the uncertainty in the probabilistic analysis is effectively reduced and the reliability level strongly increased.

## 5.3 General conclusion of the two approaches on safety and economic implications

Bearing capacity calculation of foundation is one of the most challenging problems for geotechnical engineers. The difficulty comes from multiple sources of variability and uncertainty.

PEM, FOSM, and SOSM methods were applied to evaluate the reliability of the bearing capacity. As these approaches did not provide any probability density function, one had to assume a certain distribution to plot the bearing capacity results and then estimate the corresponding failure probability by integrating over the unsafe region of the assumed density function. In this way, estimates of the failure probability are highly sensitive to the assumed distribution.

Moreover, the approximation procedures FOSM and SOSM showed some additional drawbacks in the reliability analysis of the bearing capacity problem. Bearing capacity is highly non-linear with $\tan\phi'$, which is one of the input variables that play an important role in this analysis. Because of this, FOSM and SOSM also show variability in the reliability analysis of a structure that has a highly non-linear performance function, such as a shallow foundation design. The PEM, as a direct non-iterative method, provides a value for the skewness coefficient, thus being a more promising alternative. PEM method showed to give better failure probability results by overcoming challenges of FOSM,SOSM. During design of isolated spread footing it is observed that PEM method value gives slightly higher depth of footing than Vesic results. The reinforcement cost is also higher during reliability based design. But level of confidence is high since target reliability is considered and probability of failure is well determined. Therefore, reliability based design value is more reliable than deterministic one eventhough the cost is a little bit higher. When these two approaches compared footing depth variation shows 8.25% and cost of reinforcement shows 7.67% difference.

In the other case, for pile foundation design using reliability based design input values give less no. of piles than deterministic values.. This makes it more economical. Reliabilty based methods also show failure of probabilities. Based on selected probability density distribution failurty probability can be easily observed. Especially by increasing correlation coefficients more certain bearing capacity can be obtained. One can notice that deterministic methods use more parameters to solve bearing capacity but PEM uses two soil parameters and solves the problem easily. Determinstic methods increase factor of safety to be more certain while reliability based design methods increase correlation coefficients during calculation. The first one determined based on experience and recommended values but the latter one tries to bring it with calculation and observes probability of failures. This increases level of confidence and also saves unnecessary costs. It can be concluded that, using reliability based design methods play its great role on safe and economical design of geotechnical structures.

## 5.4 Recommendations

As a recommendation for future research, it is better to perform the application of well-known reliability based design methods to other geotechnical problems like for slope stability, retaining walls and others. Combining the traditional deterministic approach and the reliability based analysis methods in further research can be beneficial to geotechnical engineering practice, supporting engineering judgement and improving the decision-making process.

As observed in this work the selected reliability based design method i.e two point estimation method has some limitations.Therefore, consider these limitations during usage of this method and also for future research works it is better to propose more advanced solutions to overcome these drawbacks.

In Ethiopian academic policies reliability based design methods are not commonly given as a course in geotechnical engineering departments. It is better to include these methods in foundation engineering courses which will help geotechnical engineers in many ways.

Last but not least, it is better to perform more researches of reliability based design methods by considering different conditions such as different types of foundations, non linear situations, lognormal functions etc.

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# APPENDICES

## Appendix A: Definition of some terms

**Reliability:** The probability that a system performs satisfactorily the intended function under specified operating conditions, during its design period.

**Random variables:** Variables whose values are not known with certainty.

**Independent** : Probabilities of occurrence of one event do not depend on another.

**Experiment** : Any process whose outcome is uncertain.

**Sample space**: The set of all possible outcomes of an experiment.

**Sample:** An individual or group, selected from a population, from whom or which data are collected.

**Cumulative frequency:** The frequency of data points that have values less than or equal to the upper bound of an interval in the frequency plot.

**Population:** encompasses the entire group for which the data are alleged to apply.

**Mean:** Arithmetic average of a distribution of scores and it provides a single, simple number that gives a rough summary of the distribution.

**Median:** The score in the distribution that marks the $50^{th}$ percentile.

**Mode:** The least used of the measures of central tendency because it provides the least amount of information. It simply indicates which score in the distribution occurs most often, or has the highest frequency.

**Range**: The difference between the largest score (the maximum value) and the smallest score (the minimum value) of a distribution.

**Interquartile Range:** The difference between the score that marks the 75th percentile (the third quartile) and the score that marks the 25th percentile (the first quartile).

**Variance:** provides a statistical average of the amount of dispersion in a distribution of scores.

**Standard Deviation:** The typical, or average, deviation between individual scores in a distribution and the mean for the distribution.

**Normal Distribution:** Also known as the Gaussian distribution is the classic bell-shaped curve that arises frequently in data sets.

**Skewness:** When a sample of scores is not normally distributed (i.e., not the bell shape), there are a variety of shapes it can assume. If there are a few scores creating an elongated tail at the higher end of the distribution, it is said to be positively skewed. If the tail is pulled out toward the lower end of the distribution, the shape is called negatively skewed.

**Kurtosis:** refers to the shape of the distribution in terms of height, or flatness. When a distribution has a peak that is higher than that found in a normal, bell-shaped distribution, it is called 'leptokurtic'. When a distribution is flatter than a normal distribution, it is called 'platykurtic'.

**Standard Error:** is the denominator in the formulae used to calculate many inferential statistics. This is because the standard error is the measure of how much random variation we would expect from samples of equal size drawn from the same population.

**Limit State(LS):** is the condition beyond which the structure or a component fail to fulfill in some way the intended purpose for which it was designed.

**Ultimate Limit State(ULS):** deals with the maximum loading capacity of the structure or element.

**Serviceability Limit State(SLS):** deals with the functionality and service requirements of a structure to ensure adequate performance under expected conditions.

## Appendix B: The derivatives of bearing capacity for the FOSM and SOSM methods

**B.1** The first derivatives of bearing capacity computed analytically with respect to soil parameters c' and tanφ' are given by equation

$$\frac{\partial qf}{\partial c'}=\frac{1}{tan\varphi'}\left\{e^{\pi tan\varphi'}[tan\varphi' + (1 + tan^2\varphi')^{\frac{1}{2}}]^2 - 1\right\}$$

$$\frac{\partial qf}{\partial tan\varphi'}=$$

$$=-22.5+ c'\left\{\frac{1}{tan^2\varphi'} - \frac{1}{tan^2\varphi'}e^{\pi tan\varphi'}[tan\varphi' + \left(1 + tan^2\varphi')^{\frac{1}{2}}\right]^2 + \frac{\pi}{tan^2\varphi'}.e^{\pi tan\varphi'}[tan\varphi' + \left(1 + tan^2\varphi')^{\frac{1}{2}}\right]^2 + 4e^{\pi tan\varphi'}\left[tan\varphi' + (1 + tan^2\varphi')^{\frac{1}{2}}\right].[1 + \frac{1}{2}(1 + tan^2\varphi')^{\frac{-1}{2}}]\right\} + q.\left\{\pi e^{\pi tan\varphi'}[tan\varphi' + \left(1 + tan^2\varphi')^{\frac{1}{2}}\right]^2 + 4e^{\pi tan\varphi'}tan\varphi'\left[tan\varphi' + (1 + tan^2\varphi')^{\frac{1}{2}}\right].[1 + \frac{1}{2}(1 + tan^2\varphi')^{\frac{-1}{2}}]\right\} + 1.5\gamma.\left\{e^{\pi tan\varphi'}[tan\varphi' + \left(1 + tan^2\varphi')^{\frac{1}{2}}\right]^2 + \pi tan\varphi' e^{\pi tan\varphi'}[tan\varphi' + \left(1 + tan^2\varphi')^{\frac{1}{2}}\right]^2 + 4tan^2\varphi' e^{\pi tan\varphi'}\left[tan\varphi' + (1 + tan^2\varphi')^{\frac{1}{2}}\right].[1 + \frac{1}{2}(1 + tan^2\varphi')^{\frac{-1}{2}}]\right\}$$

**B.2** The second derivatives of bearing capacity computed analytically with respect to soil parameters c' and tanφ' are given by equation

$\frac{\partial^2 qf}{\partial^2 c'} = 0$

$\frac{\partial^2 qf}{\partial^2 tan\varphi'} =$

$$c'e^{\pi tan\varphi'}\left[tan\varphi' + (1+tan^2\varphi')^{\frac{1}{2}}\right].\left\{\frac{2}{tan^3\varphi'}\left[tan\varphi' + (1+tan^2\varphi')^{\frac{1}{2}}\right] - \frac{\pi}{tan^2\varphi'}.\left[tan\varphi' + (1+tan^2\varphi')^{\frac{1}{2}}\right] - \frac{4}{tan\varphi'}[1+\frac{1}{2}\left(1+tan^2\varphi'\right)^{\frac{-1}{2}}\right] + \pi[-\frac{1}{tan^2\varphi'}\left[tan\varphi' + (1+tan^2\varphi')^{\frac{1}{2}}\right] + \frac{\pi}{tan\varphi'}\left[tan\varphi' + (1+tan^2\varphi')^{\frac{1}{2}}\right] + 4[1+\frac{1}{2}\left(1+tan^2\varphi'\right)^{\frac{-1}{2}}\right] + 4[\pi[1+\frac{1}{2}\left(1+tan^2\varphi'\right)^{\frac{-1}{2}}\right] + 2tan\varphi'[1+\frac{1}{2}\left(1+tan^2\varphi'\right)^{\frac{-1}{2}}\right]^2 + tan\varphi'\left[\frac{1}{2}(1+tan^2\varphi')^{\frac{-3}{2}}\right]]\right\} + q\left\{\pi e^{\pi tan\varphi'}[\pi[tan\varphi' + \left(1+tan^2\varphi'\right)^{\frac{1}{2}}\right]^2 + 4tan\varphi'\left[tan\varphi' + (1+tan^2\varphi')^{\frac{1}{2}}\right].\left[1+\frac{1}{2}\left(1+tan^2\varphi'\right)^{\frac{-1}{2}}\right]\right] + 4[\pi e^{\pi tan\varphi'}tan\varphi'\left[tan\varphi' + (1+tan^2\varphi')^{\frac{1}{2}}\right].[1+\frac{1}{2}\left(1+tan^2\varphi'\right)^{\frac{-1}{2}}\right] + e^{\pi tan\varphi'}\left[2tan\varphi' + (1+tan^2\varphi')^{\frac{1}{2}}\right] + tan^2\varphi'\left[(1+tan^2\varphi')^{\frac{-1}{2}}\right][1+\frac{1}{2}\left(1+tan^2\varphi'\right)^{\frac{-1}{2}}\right] + tan^2\varphi' e^{\pi tan\varphi'}\left[-\frac{1}{2}(1+tan^2\varphi')^{\frac{-3}{2}}\right]\left[tan\varphi' + (1+tan^2\varphi')^{\frac{1}{2}}\right]\right\} + 22.5e^{\pi tan\varphi'}.\left\{\pi[tan\varphi' + \left(1+tan^2\varphi'\right)^{\frac{1}{2}}\right]^2 + 4tan\varphi'\left[tan\varphi' + (1+tan^2\varphi')^{\frac{1}{2}}\right][1+\frac{1}{2}\left(1+tan^2\varphi'\right)^{\frac{-1}{2}}\right] + \pi[tan\varphi' + \left(1+tan^2\varphi'\right)^{\frac{1}{2}}\right]^2 + \pi tan\varphi'[tan\varphi' + \left(1+tan^2\varphi'\right)^{\frac{1}{2}}\right]^2 + 4tan^2\varphi'\left[tan\varphi' + (1+tan^2\varphi')^{\frac{1}{2}}\right][1+\frac{1}{2}\left(1+tan^2\varphi'\right)^{\frac{-1}{2}}\right] + 4tan\varphi'[\pi tan\varphi'\left[tan\varphi' + (1+tan^2\varphi')^{\frac{1}{2}}\right][1+\frac{1}{2}\left(1+tan^2\varphi'\right)^{\frac{1}{2}}\right] + [1+\frac{1}{2}\left(1+tan^2\varphi'\right)^{\frac{-1}{2}}\right][tan\varphi' + (1+tan^2\varphi')^{\frac{1}{2}} + tan^2\varphi'\left[(1+tan^2\varphi')^{\frac{-1}{2}}\right] + tan^2\varphi'\left[tan\varphi' + (1+tan^2\varphi')^{\frac{1}{2}}\right].[1+\frac{1}{2}\left(1+tan^2\varphi'\right)^{\frac{-1}{2}}\right]]\right\}$$

The second derivatives of bearing capacity computed analytically with respect to both soil parameters c' and tanφ' are given by equation

$$\frac{\partial^2 qf}{\partial \tan\varphi' \, \partial c'} =$$

$$=\left\{\frac{1}{\tan^2\varphi'} - \frac{1}{\tan^2\varphi'} e^{\pi\tan\varphi'}\left[\tan\varphi' + \left(1+\tan^2\varphi'\right)^{\frac{1}{2}}\right]^2 + \frac{\pi}{\tan\varphi'} e^{\pi\tan\varphi'}\left[\tan\varphi' + \left(1+\tan^2\varphi'\right)^{\frac{1}{2}}\right]^2 + 4e^{\pi\tan\varphi'}\left[\tan\varphi' + (1+\tan^2\varphi')^{\frac{1}{2}}\right]\left[1+\frac{1}{2}\left(1+\tan^2\varphi'\right)^{\frac{-1}{2}}\right]\right\}$$

## Appendix C: Abbreviations

RBD = Reliability Based Design

MCS = Monte Carlo simulation

FOSM = First Order Second Moment

SOSM = Second Order Second Moment

PEM = Point Estimation Method

LRFD = Load and Resistance Factor Design

ASD = Allowable Stress Design

COV = Coefficient of variation

FS = Factor of Safety

BF = Both Faces

# Appendix D : Details for concrete and steel properties

**For C25 and S460**

For C25

$$fcd = \frac{0.85*fck}{1.5} = 0.85*\frac{25}{\frac{1.25}{1.5}} = 11.33\text{MPa}.$$

$$fctd = fbd = \frac{0.21*fck^{\ 2/3}}{1.5} = \frac{0.21*(\frac{25}{1.25})^{\ 2/3}}{1.5} = 1.03\text{MPa}$$

For S460

$$fyd = \frac{fy}{1.15} = \frac{460}{1.15} = 400\text{MPa}$$

$$\rho min = \frac{0.5}{\text{fy}} = \frac{0.5}{460} = 0.0010869$$

$$\rho x = \rho y = [1-(1-\frac{2M}{fcd*b*d^{\ 2}})^{\ 1/2}]*\ \frac{fcd}{\text{fyd}}$$

$$\rho x = \rho y = [1-(1-\frac{2*70}{11.33*1000^{\ 3}*1*0.86^{\ 2}})^{\ 1/2}]*\ \frac{11.33}{400}$$

ρx = ρy = 0.0002376 < ρmin

Therefore, take ρe = ρmin = **0.0010869**

## Appendix E : Additional calculation results of PEM

### A) Case I

| | | PEM CALCULATION | |
|---|---|---|---|
| SAMPLE 1 LEFT | | RIGHT | |
| Parameters | Value | Parameters | Value |
| Df(m) | 2 | Df(m) | 2 |
| B(m) | 2.5 | B(m) | 2.5 |
| γ(KN/m3) | 12.09 | γ(KN/m3) | 12.09 |
| C'(KPa) | 8.5 | C'(KPa) | 8.5 |
| ϕ′(degrees) | 21.13 | ϕ′(degrees) | 26.74 |
| tanϕ′ | 0.3865 | tanϕ′ | 0.5039 |
| Nq | 8.43 | Nq | 15.542 |
| Nc | 19.15 | Nc | 28.76 |
| Nγ | 5.89 | Nγ | 12.756 |
| qo | 24.18 | qo | 24.18 |
| | | | |
| OUTPUT | | OUTPUT | |
| qu | 455.625025 | qu | 813.04061 |

### B) Case II

| **tanϕ′+C'+(++)** | | **tanϕ′+C'-** | |
|---|---|---|---|
| Parameters | Value | Parameters | Value |
| Df(m) | 2 | Df(m) | 2 |
| B(m) | 2.5 | B(m) | 2.5 |
| γ(KN/m3) | 12.09 | γ(KN/m3) | 12.09 |
| C'(KPa) | 17.85 | C'(KPa) | 6.57 |
| ϕ′(degrees) | 26.74 | ϕ′(degrees) | 26.74 |
| tanϕ′ | 0.5039 | tanϕ′ | 0.5039 |
| Nq | 15.542 | Nq | 15.542 |
| Nc | 28.76 | Nc | 28.76 |
| Nγ | 12.756 | Nγ | 12.756 |
| qo | 24.18 | qo | 24.18 |
| | | | |
| OUTPUT | | OUTPUT | |
| qu | 1081.947 | qu | 757.5338 |

**tanϕ′-C'+**

| Parameters | Value |
|---|---|
| Df(m) | 2 |
| B(m) | 2.5 |
| γ(KN/m3) | 12.09 |
| C'(KPa) | 17.85 |
| ϕ′(degrees) | 21.13 |
| tanϕ′ | 0.3865 |
| Nq | 8.43 |
| Nc | 19.15 |
| Nγ | 5.89 |
| qo | 24.18 |
| | |
| OUTPUT | |
| qu | 634.6775 |

**tanϕ′-C'-**

| Parameters | Value |
|---|---|
| Df(m) | 2 |
| B(m) | 2.5 |
| γ(KN/m3) | 12.09 |
| C'(KPa) | 6.57 |
| ϕ′(degrees) | 21.13 |
| tanϕ′ | 0.3865 |
| Nq | 8.43 |
| Nc | 19.15 |
| Nγ | 5.89 |
| qo | 24.18 |
| | |
| OUTPUT | |
| qu | 418.6655 |

**C) Case III**

**tanϕ′+C'+(++)**

| Parameters | Value |
|---|---|
| Df(m) | 2 |
| B(m) | 2.5 |
| γ(KN/m3) | 12.09 |
| C'(KPa) | 36.12 |
| ϕ′(degrees) | 26.74 |
| tanϕ′ | 0.5039 |
| Nq | 15.542 |
| Nc | 28.76 |
| Nγ | 12.756 |
| qo | 24.18 |
| | |
| OUTPUT | |
| qu | 1607.392 |

**tanϕ′+C'-(+ -)**

| Parameters | Value |
|---|---|
| Df(m) | 2 |
| B(m) | 2.5 |
| γ(KN/m3) | 12.09 |
| C'(KPa) | 5.43 |
| ϕ′(degrees) | 26.74 |
| tanϕ′ | 0.5039 |
| Nq | 15.542 |
| Nc | 28.76 |
| Nγ | 12.756 |
| qo | 24.18 |
| | |
| OUTPUT | |
| qu | 724.7474 |

**tanϕ′-C'+(-+)**

| Parameters | Value |
|---|---|
| Df(m) | 2 |
| B(m) | 2.5 |
| γ(KN/m3) | 12.09 |
| C'(KPa) | 36.12 |
| ϕ′(degrees) | 21.13 |
| tanϕ′ | 0.3865 |
| Nq | 8.43 |
| Nc | 19.15 |
| Nγ | 5.89 |
| qo | 24.18 |

OUTPUT

| | |
|---|---|
| qu | 984.548 |

**tanϕ′-C'-(- -)**

| Parameters | Value |
|---|---|
| Df(m) | 2 |
| B(m) | 2.5 |
| γ(KN/m3) | 12.09 |
| C'(KPa) | 5.43 |
| ϕ′(degrees) | 21.13 |
| tanϕ′ | 0.5139 |
| Nq | 8.43 |
| Nc | 19.15 |
| Nγ | 5.89 |
| qo | 24.18 |

OUTPUT

| | |
|---|---|
| qu | 396.8345 |